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COLLEGE OF ENGINEERING DEPARTMENT OF ELECTRICAL ENGINEERING SPACE PHYSICS RESEARCH LABORATORY

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The Lunar Atmosphere

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THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
Department of Electrical Engineering
Space Physics Research Laboratory

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THE LUNAR ATMOSPHERE

F. Hinton
D. R. Taeusch

ORA Project 03555

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NOMENCLATURE

A	atomic weight
B	magnetic field in gauss
C	root-mean-square thermal speed of particle
E	kinetic energy of a particle
E_F	kinetic energy of incident particle after collision
E_{flow}	kinetic energy of a particle at the mean velocity of the flow in the solar wind
E_T	total energy of a particle, kinetic plus potential
G	universal gravitational constant
H	Hamiltonian function for a particle
I_1, I_2, I_3	escape and production mechanism integrals
J	solar wind flux in particles/cm ² /sec
J_J	Jeans evaporation flux
J_p	proton flux in the solar wind
J_s	source strength in particles/cm ² /sec
M	mass of moon
N	total number of particles in the lunar atmosphere
N_c	total number of particles in a square centimeter column
N_i	total number of ions in the lunar atmosphere
$N(t)$	number of atoms produced per cm ² of lunar surface in a time t by radioactive decay processes
P	magnitude of angular momentum of a particle

NOMENCLATURE (Continued)

S	volume production rate of particles
T	temperature; °K
T_g	temperature of gauge
T_m	temperature of moon
V	scale volume
Z	charge of an ion in units of the electron charge
$a(t)$	activity in disintegrations/sec/cm ²
b	impact parameter
c	speed of particles
\bar{c}	mean speed of particles
e	electron charge in esu
f	molecular distribution function
g	acceleration of gravity at the moon's surface
h	scale height for an isothermal atmosphere
k	Boltzmann's constant
m	mass of a particle in grams
m_e	mass of electron
m_p	mass of proton
m_1	mass of incident particle in a collision
m_2	mass of target particle in a collision
n	number density of particles in particles/cm ³
n_g	number density of particles inside gauge

NOMENCLATURE (Continued)

n_0	number density of particles in the lunar atmosphere near the surface
n_{oi}	number density of positive ions near the surface
n_∞	number density of particles in interplanetary space
n_{sg}	number density of particles inside gauge which originated in the solar wind
p_i	momentum conjugate to the i th position coordinate
q_i	i th position coordinate of a particle
r	distance from the center of the moon
r_0	radius of the moon
t_c	"self-collision time" for particles in a plasma
t_{eq}	time of equipartition for particles in a plasma
t_f	time of flight of a particle
$t_{1/2}$	half-life for radioactive decay
u	velocity of particles
$\langle u \rangle_{avg}$	mean velocity of particles
v_{esc}	escape velocity at the lunar surface
v_{orb}	orbital velocity at the lunar surface
α	angle between the velocity vector of a particle and the vector from the center of the moon, or the vertical direction
β	the fraction of the particles in the lunar atmosphere which are exposed to the sun
γ	the geometric probability for escape of a particle from the atmosphere
$d\Omega$	"volume" element in phase space

NOMENCLATURE (Concluded)

η	angle between the centerline of the orifice and the sun
Θ	angle of scattering in center-of-mass coordinates
θ	angle of scattering of incident particle
λ	mean-free-path of particle
λ_{90}	mean path for deflection by 90°
πa^2	gas kinetic cross-section of a particle, calculated from measured transport properties
ρ	range of a particle on the moons surface
$\Sigma\Phi_0$	photoionization or photodissociation probability per sec
$\sigma^{(cx)}$	measured charge exchange cross section
$\sigma^{(el)}$	measured elastic collision cross section with a charged particle
$\sigma_{esc}^{(cx)}$	charge exchange "escape cross section"
$\sigma_{esc}^{(el)}$	elastic "escape cross section"
$i\sigma_{esc}^{(el)}$	elastic "escape cross section" for a positive ion
$\sigma(\Theta)$	differential scattering cross section
τ	mean-free-time of a particle
ϕ	angle of scattering of target particle in a collision with an incident particle
ψ	angular range of a particle on the moons surface
Ψ	angle used in describing the trajectory of a particle, measured from the apogee

CONSTANTS

Earth:

Radius: 6,371 km
Mass: 5.975×10^{24} kg
Mean Distance from Sun: 1.495×10^8 km
Mean Velocity in Orbit Around Sun: 29.76 km/sec

Moon:

Radius: 1,739.3 km
Mass: 7.343×10^{22} kg
Mean Distance from Earth: 3.844×10^5 km
Mean Velocity in Orbit Around Earth: 1.03 km/sec
Surface Gravity: 161.9 cm/sec²
Escape Velocity at Surface: 2.37 km/sec
Orbital Velocity at Surface: 1.68 km/sec

Universal Gravitation Constant: $G = 6.670 \times 10^{-8}$ dyne cm²/gram²

One Electron Volt = 1.602×10^{-12} erg = $k(11,606^\circ\text{K})$

Boltzmann's Constant: $k = 1.380 \times 10^{-16}$ erg/°K

Avagadro's Number: 6.025×10^{23} mole⁻¹

Proton Mass: $m_p = 1.660 \times 10^{-24}$ grams

Electron Mass: $m_e = 9.108 \times 10^{-28}$ grams ($m_p/m_e = 1836$)

Electron Charge: 4.803×10^{-10} esu.

SECTION I

INTRODUCTION

At present, this laboratory, in cooperation with the Goddard Space Flight Center, Aeronomy and Meteorology Division, is under contract with NASA to design, construct, test and calibrate an instrumented package capable of measuring certain atmospheric parameters on the moon's surface for the purpose of determining the lunar atmospheric structure. This package is to be flown on one or more of the Surveyor soft-landing lunar vehicles to be launched in 1965.

This report is the result of a study conducted to obtain information about known facts and theories concerning the lunar atmosphere and to postulate a theory of its structure. The problem that presents itself here is that the moon's surface atmosphere is almost certainly an exosphere, whose characteristics are determined not only by the moon's presence and gaseous contribution itself but also by the effect of the solar wind on the surface materials and gaseous envelope. Since the solar wind itself is not well understood, theoretical determination of the structure of a lunar atmosphere becomes complex, and at best only order-of-magnitude effects of the various known inputs can be determined by theoretical means. The over-all effect of all inputs on the lunar atmosphere has been a subject of much debate and will be known only when we can measure experimentally the parameters, such as neutral number densities of the various constituents, ion number density and particle temperatures.

Section II of this report delineates the limited experimental and observational results to date of measurements of the physical properties of the lunar atmosphere, "solar wind," lunar magnetic field and lunar surface temperature. In Section III, the general concepts of the lunar atmosphere are discussed with reference to the mechanisms which affect an exosphere. A simple model of the lunar atmosphere is described in Section IV. Section V describes the experimental measurements to be made by this laboratory, the expected problems and the meaning of the expected results. Since the period between July of 1964 and July of 1965 is expected to be a period of minimum solar activity, we will be concerned here primarily with conditions during "quiet sun."

SECTION II

A SURVEY OF EXPERIMENTAL AND OBSERVATIONAL RESULTS

A. THE LUNAR ATMOSPHERE

For a long time, astronomers attempted to find some evidence of an atmosphere on the moon, but their results showed that the lunar atmosphere is practically non-existent. A summary of the attempts to find evidence of the lunar atmosphere by using refraction phenomena, spectroscopic methods, and the brightness and polarization of the scattered light has been given by Sytinskaya.¹ Here, we will only mention the measurements of the French astronomer Dollfus,² which have been the most sensitive attempt at an optical determination of the density of the lunar atmosphere. Dollfus' results do not give a positive determination of the density; rather, an upper limit for the density is set by the sensitivity of his apparatus. In measuring the polarization of the scattered light from the moon, Dollfus showed that the density of the lunar atmosphere is less than 10^{-9} of the density of the terrestrial atmosphere at sea level, which corresponds to a number density of about 10^{10} particles/cm³.

Similarly, an upper limit for the density of electrons in the lunar atmosphere at the surface was derived by Elsmore.³ On the basis of a refraction of radio waves of 3.7 meter wavelength estimated from the difference between the observed and calculated times of obscuration for the lunar occultation of the Crab Nebula at Cambridge on Jan. 24, 1956, Elsmore deduced an electron density of about 10^3 /cm³ in excess of that of the surrounding interplanetary medium. He estimates that the density of the atmosphere at the surface of the moon is about 2×10^{-13} of that of the earth's atmosphere at normal temperature and pressure, or about 5×10^6 particles/cm³. (It is interesting to note in this connection that Jan., 1956 appeared approximately halfway between sunspot minimum and sunspot maximum. Thus, the density during quiet sun, or sunspot minimum, would be expected to be somewhat different. This will be discussed in Section IV.)

Evidence of an increased electron density in the vicinity of the moon has also been given by Krasovskii.⁴ On the basis of ion trap measurements performed on Soviet cosmic rockets, he states that the density of ionized particles at a distance of some radii from the earth is of the order of several thousand ion pairs/cm³ or less, and that somewhat higher currents between the electrodes of an ion trap were observed near the moon.

B. THE SOLAR WIND AND THE INTERPLANETARY MATERIAL

Because the lunar atmosphere is so tenuous we expect that the interplan-

etary medium may have a considerable effect on its structure. It appears that the moon's surface cannot possibly be shielded by its atmosphere from the interplanetary medium or the solar radiation. In addition to the electromagnetic radiation, the sun also emits radiation in the form of charged particles. As shown by Herring and Licht,⁵ these have a considerable effect on the lunar atmosphere. The following is a summary of the evidence concerning the corpuscular radiation, which has been called by Parker⁶ the "solar wind."

Based on the observation of the acceleration of Type I comet tails, Biermann⁷ has presented evidence for the presence of an outward streaming component of ionized gas from the sun. He suggests that the solar corpuscular radiation consists of ions and electrons in the same number, about 10^3 to 10^5 /cm³, which are emitted from the sun with ordered velocities of about 1000 km/sec. In a discussion of Biermann's paper, Kiepenheuer⁸ has derived a lower limit for the particle density of about 6/cm³ assuming a velocity of 1000 km/sec. In a series of papers⁶ Parker summarized various geomagnetic effects which are assumed to be associated with discrete solar streams from an active sun. From these considerations, densities for quietest conditions appear to be at least 100 ions/cm³ with a minimum velocity of 500 km/sec. Parker⁹ asserts that the corpuscular radiation represents an outward streaming of matter from the sun that is hydrodynamic in character: it consists of an electrically neutral plasma in which the mean-free-path for interparticle collisions is small compared with the dimensions of the flow. Using the hydrodynamic equations, Parker shows that the solar corona must expand, and he believes that the interplanetary material and the solar corpuscular radiation are both merely the hydrodynamic continuation of the expanding solar corona.

The density of the interplanetary material has also been estimated from the results of optical measurements. In Feb. and March, 1952, Behr and Siedentopf¹⁰ observed the brightness and the polarization of the evening zodiacal light. (This was a time of relatively quiet sun, since sunspot minimum occurred in 1954.) After examining the polarized component of the light, they concluded that the electron density in the vicinity of the earth is about 600/cm³, assuming that the scattering of light by free electrons is entirely responsible for the polarized component. However, Blackwell and Ingham¹¹ have concluded from their data that the scattering is almost entirely due to dust and that it is unlikely that the electron density in the vicinity of the earth exceeds 120/cm³.

Direct measurements of corpuscular fluxes have been reported by Gringauz.¹² He states that during the last part of the trajectory of Lunik II, positive collector currents in ion traps were recorded which correspond to positive ion fluxes of about 2×10^8 ions/cm² sec. (Geomagnetic disturbances during this portion of the flight were characterized by a K-index of 5, which indicates a moderate disturbance level.) During the flight of space probe IV launched in the direction of Venus in Feb., 1961, currents were recorded which correspond to a density of positive corpuscular flux of about 10^9 ions/cm² sec. (Shortly

after these readings were taken, the commencement of a magnetic storm with amplitude of perturbation of the horizontal component of the geomagnetic field of 10^{-3} gauss = 100γ was observed, an average magnetic storm which is consistent with a K-index of 5.) Both of the Russian results would be expected to give fluxes greater than those expected during quiet sun. However, these results are smaller than Parker's estimate of the minimum flux for quietest conditions of 5×10^9 ions/cm² sec corresponding to a density of 100/cm³ and a velocity of 500 km/sec.

C. THE SELENOMAGNETIC FIELD

Since the solar wind is expected to have a considerable effect on the lunar atmosphere, and the stream of charged particles will be affected by any magnetic field, it is of interest to know the magnitude of the magnetic field of the moon. A magnetic field would shield or markedly change the influence of the solar wind upon the lunar surface and atmosphere. A criterion for shielding has been given by Parker¹³ as

$$\frac{1}{2} n m \langle u \rangle_{\text{avg}}^2 = \frac{B^2}{8\pi}.$$

It will be seen in Section III-C that a good estimate for the density of the ordered translational kinetic energy in the solar wind is about 5×10^{-8} ergs/cm³. The field required by the above formula for shielding is therefore $B = 1.12 \times 10^{-3}$ gauss = 112γ .

The Russian scientists Dolginov, et al.,¹⁴ have reported an attempt to measure the magnetic field of the moon. They state that the magnetometer aboard Space Rocket II did not detect a magnetic field up to 50 km from the lunar surface, and that if the moon had a magnetic field whose intensity at the lunar surface exceeded 100γ , it would have been detected. Therefore, it appears that the moon is not effectively shielded from the solar wind by its magnetic field. However, it is possible that the lunar magnetic field itself is greater than 100γ , but that the solar corpuscular radiation, deflected in that field, creates a current having a magnetic field which tends to cancel the lunar field. This has been discussed by Neugebauer.¹⁵

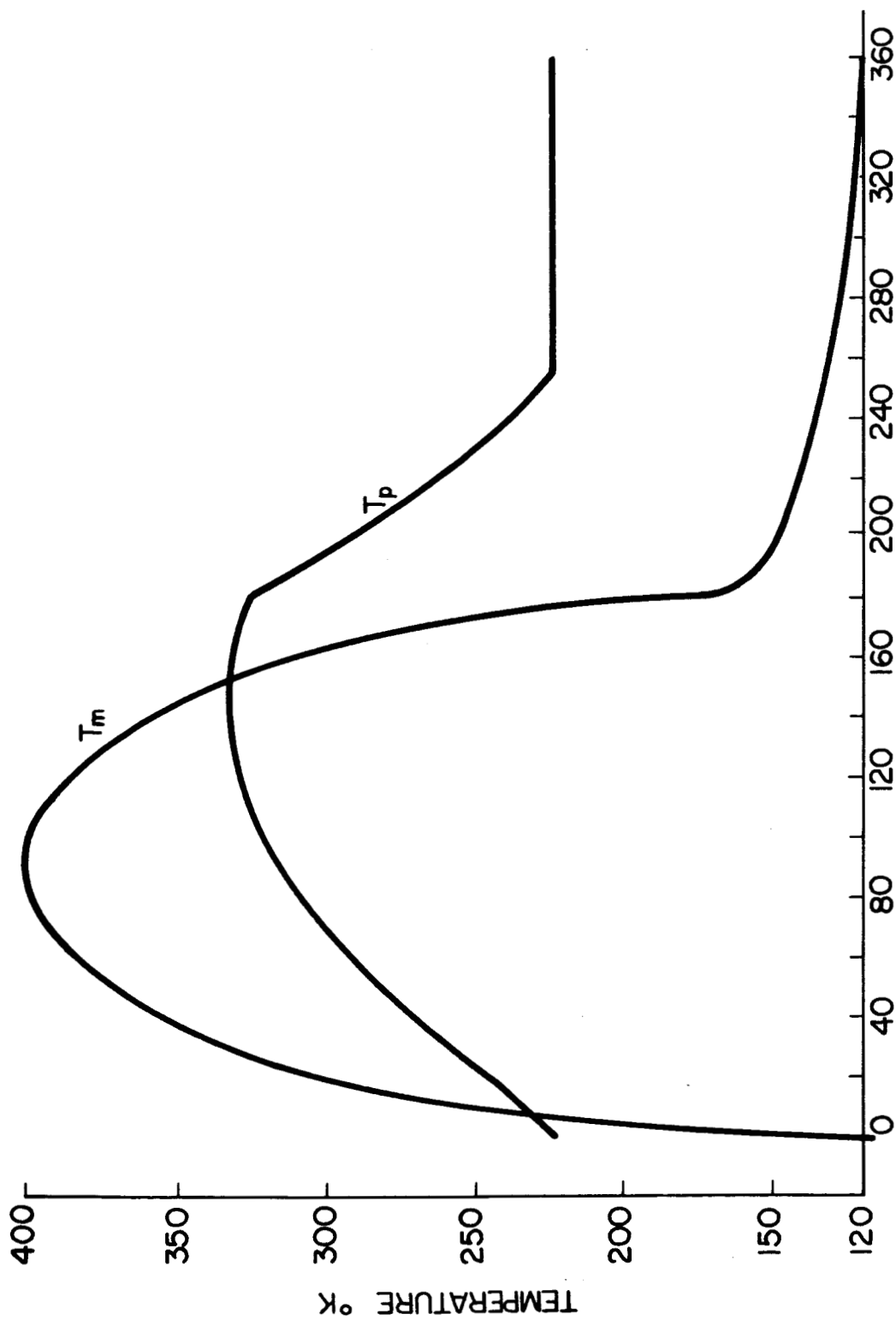
D. THE LUNAR SURFACE TEMPERATURE

The structure of the lunar atmosphere will be greatly affected by the surface temperature of the moon. High surface temperatures will increase the escape of molecules from the vicinity of the moon because of the high velocities which they can acquire upon contact with the surface. Low surface temperatures can cause condensation of atmospheric volatiles on the lunar

surface if their partial pressures exceed their vapor pressures at that temperature. It is therefore of interest to know the variation of temperature on the surface.

Measurements of the lunar surface temperature were made by Pettit and Nicholson¹⁶ in 1927 and by Pettit¹⁷ in 1939. Although these measurements were made over twenty years ago, they are still the most valuable measurements which we have of the lunar surface temperature. During the eclipse of June 14, 1927, Pettit and Nicholson determined the temperature of a small area near the border of the lunar disc. With an improved technique Pettit observed the temperature of a small area near the center of the lunar disc during the eclipse of October 28, 1939. Pettit and Nicholson found the temperature of an area opposite to the sun to be 120°K , with an uncertainty of 13% corresponding to the probable error of 50% in the recorded amount of radiation. The temperature near the center of the lunar disc at full moon, as found by Pettit just before the beginning of the 1939 eclipse, was 370°K . These observations were discussed by Wesselink,¹⁸ who gave a predicted curve of the variation in surface temperature of an area near the center of the lunar disc during a lunation. Given in Fig. 1 is a curve similar to Wesselink's except that it has been shifted by about 30°K , so that the maximum temperature is 400°K . This curve has been used in the thermal design of the instrument package as stipulated by the Jet Propulsion Laboratory. It can be seen from this curve that a reasonable average temperature for the sunlit side is about 300°K and a reasonable average temperature for the night side is about 130°K .

The curve labeled T_p is the expected variation in the instrument package temperature which is discussed in Section V.



ANGLE OF ROTATION WRT SUN

Fig. 1. Assumed equatorial moon surface temperature (T_m) and instrument package temperature (T_p) vs. suns angular position. 0° at night to day terminator.

SECTION III

SOME GENERAL IDEAS CONCERNING THE LUNAR ATMOSPHERE

In this section, we discuss the equations governing the density distribution in an exosphere, and an isothermal atmosphere, the possible sources of gas for the lunar atmosphere, the effect of the solar wind on the lunar atmosphere and the molecular escape mechanisms. In all cases, it will be assumed that there are no collisions between neutral particles, since their mean-free path is long compared with their ballistic trajectory. The so-called base of the exosphere is, therefore, taken to be at the lunar surface. This and other assumptions are discussed in relation to their use in the presentation of the model lunar atmosphere in Section IV.

A. THE MOLECULAR DISTRIBUTION FUNCTION AND LIOUVILLE'S THEOREM

We could determine the structure of the lunar atmosphere if we knew, at any given time, the positions and states of motion of all of the particles up to a great height above the lunar surface. For a given particle, we know the necessary information if we are given, as functions of time, its three position coordinates, q_1, q_2, q_3 , and their three conjugate momenta, p_1, p_2, p_3 . The motion of the particle is described by the solution of the equations of motion

$$\dot{q}_s = \frac{\partial H}{\partial p_s}, \quad \dot{p}_s = - \frac{\partial H}{\partial q_s}, \quad s = 1, 2, 3, \quad (3-1)$$

subject to certain initial conditions, where H is the Hamiltonian function for that particle. If the particle is not charged, and if we can neglect collisions between the particles, the Hamiltonian function in terms of rectangular coordinates with origin at the center of the moon is

$$H(x, y, z, p_x, p_y, p_z) = \frac{1}{2m} (p_x^2 + p_y^2 + p_z^2) - \frac{mMG}{\sqrt{x^2 + y^2 + z^2}}, \quad (3-2)$$

for

$$\sqrt{x^2 + y^2 + z^2} > r_0.$$

Since collisions are to be neglected, the initial conditions for the motion need to be specified only after an encounter with a solar wind particle or

after the particle strikes the lunar surface. The motion of the particle may be represented by the path of a point in the six-dimensional space whose coordinates are $q_1, q_2, q_3, p_1, p_2, p_3$, the phase space of the particle. Through any given point in phase space, perhaps given by the initial conditions, there passes a uniquely determined path for the representative point of a particle.

Since the determination of the motions of the large number of particles in the atmosphere is practically impossible, we must introduce some statistical postulates. These are discussed by Present.¹⁹ Here, we assume the existence of a function $f(q_1, q_2, q_3, p_1, p_2, p_3, t)$, such that $fdq_1 \dots dp_3$ is the number of particles at time t whose motions can be represented by points in phase space located in the element $d\Omega$ with boundaries $q_1, q_1+dq_1, \dots p_3, p_3+dp_3$, and having extension $dq_1 \dots dp_3$. The function f is called the molecular distribution function and represents the density of the representative points in phase space. Its use is illustrated by the way in which we find the spacial density and temperature. The density is given by

$$n(q_1, q_2, q_3) = \iiint f dp_1 dp_2 dp_3, \quad (3-3)$$

where the integration is carried over the limits of the momenta. The so-called kinetic temperature T is defined by

$$\frac{3}{2} kT \equiv \frac{1}{2} mC^2, \quad (3-4)$$

where C is the root-mean-square speed:

$$C^2 \equiv \frac{1}{n} \iiint c^2 f dp_1 dp_2 dp_3 \quad (3-5)$$

and $f dp_1 dp_2 dp_3$ is expressed in terms of c , the particle speed.

The question arises as to how one can find f . It can be shown to be the solution of a partial differential equation, as follows: imagine a cloud of representative points in phase space whose density at any point at a given time is f . The cloud will move in phase space, as each representative point traverses its dynamical path, and will pass through a stationary element $d\Omega$. The representative points crossing the face of $d\Omega$ normal to the p_1 direction, which has area $dq_1 \dots dp_3 / dp_1$, have a component of velocity, in phase space, \dot{p}_1 normal to that face. The rate of increase in the number $f dq_1 \dots dp_3$ of representative points in $d\Omega$ due to motion across this face is

$$f \dot{p}_1 \left[\frac{dq_1 \dots dp_3}{dp_1} \right]_{p_1}.$$

There is a loss

$$f \dot{p}_1 \left[\frac{dq_1 \cdots dp_3}{dp_1} \right]_{p_1+dp_1}$$

due to motion across the opposite face, and so the net increase is:

$$\left\{ f \dot{p}_1 \right\}_{p_1} - f \dot{p}_1 \left[\frac{dq_1 \cdots dp_3}{dp_1} \right]_{p_1+dp_1} = - \frac{\partial}{\partial p_1} (f \dot{p}_1) dq_1 \cdots dp_3 . \quad (3-6)$$

Summing over all the six pairs of faces, the net increase in the number of representative points in $d\Omega$ is

$$- \sum_{s=1}^3 \left[\frac{\partial}{\partial q_s} (f \dot{q}_s) + \frac{\partial}{\partial p_s} (f \dot{p}_s) \right] dq_1 \cdots dp_3 .$$

This is also equal to the rate of increase of f in the fixed element $d\Omega$,

$$\frac{\partial f}{\partial t} dq_1 \cdots dp_3 .$$

We may write, therefore,

$$\frac{\partial f}{\partial t} + \sum_{s=1}^3 \left[\frac{\partial}{\partial q_s} (f \dot{q}_s) + \frac{\partial}{\partial p_s} (f \dot{p}_s) \right] = 0 . \quad (3-7)$$

Using the equations of motion, this may be written

$$\frac{\partial f}{\partial t} + \sum_{s=1}^3 \left[\frac{\partial f}{\partial q_s} \dot{q}_s + \frac{\partial f}{\partial p_s} \dot{p}_s \right] = 0 . \quad (3-8)$$

But this simply says that the substantial derivative of f in phase space vanishes:

$$\frac{Df}{Dt} = 0 . \quad (3-9)$$

Thus, the density of representative points remains unaltered as the motion progresses, which is the statement of Liouville's theorem.

The general solution of the above partial differential equation is

$$f(q_1, \dots, p_3) = F[I_1(q_1, \dots, p_3), \dots, I_6(q_1, \dots, p_3)] \quad (3-10)$$

where F is any arbitrary function and the I 's are six integrals of the motion, which may be the initial conditions, and they may include constants of the motion such as the Hamiltonian and the angular momentum. To show this, substitute into the equation:

$$\frac{Df}{Dt} = \frac{DF}{Dt} = \sum_{i=1}^6 \frac{\partial F}{\partial I_i} \dot{I}_i. \quad (3-11)$$

This is identically zero because the integrals of the motion are defined by $\dot{I}_i \equiv 0$. The general solution can be used if we have enough additional information about the function f to determine it uniquely. This is equivalent to specifying the distribution of initial conditions, that is, by giving the function f over the entire phase space at some particular time. If the particles all begin their trajectories at the surface of the moon, then the specification of the function f at the lunar surface at some time will be sufficient. In Sections III-C and III-D this is done for special cases.

B. THE MIGRATION OF MOLECULES OVER THE LUNAR SURFACE

The picture that we envisage of the lunar atmosphere is that of particles traveling without collisions from the time they leave the surface of the moon until the time they either return to the surface or are lost into interplanetary space. We would like to know just how far a given particle will travel over the moon's surface, on the average, and what the length of time is during its flight. This will give us an idea of how fast the particles are able to migrate from one place on the moon to another.

Let us assume that a particle leaves the surface of the moon at an angle $\alpha_0 = 45^\circ$ with the vertical direction and with a speed equal to the mean speed \bar{c} of the distribution of velocities characteristic of the temperature T . If we use the Maxwellian distribution formula, this is

$$\bar{c} = \sqrt{\frac{8kT}{\pi m}}. \quad (3-12)$$

(The values of \bar{c} for various gases are listed in Table 1.) We now want to calculate the range as a function of T for various gases. For the lighter gases, such as hydrogen and helium, the range will be comparable in magnitude

to the lunar dimensions. In spherical coordinates, the law of motion for a particle near the moon with a total energy E_T and an angular momentum with magnitude P is given as follows (see Goldstein²⁰):

$$\frac{1}{r} = \frac{m^2 MG}{P^2} - \sqrt{\left(\frac{m^2 MG}{P^2}\right)^2 + \frac{2mE_T}{P^2}} \cos \psi. \quad (3-13)$$

In this equation, ψ is measured from the apogee of the orbit (see Fig. 2). For a particle which leaves the surface $r = r_0$ with a speed \bar{c} at an angle α_0 with the vertical, the energy and angular momentum constants are given as follows:

$$\frac{E_T}{m} = \frac{\bar{c}^2}{2} - \frac{MG}{r_0}, \quad (3-14)$$

$$\frac{P}{m} = \bar{c} r_0 \sin \alpha_0. \quad (3-15)$$

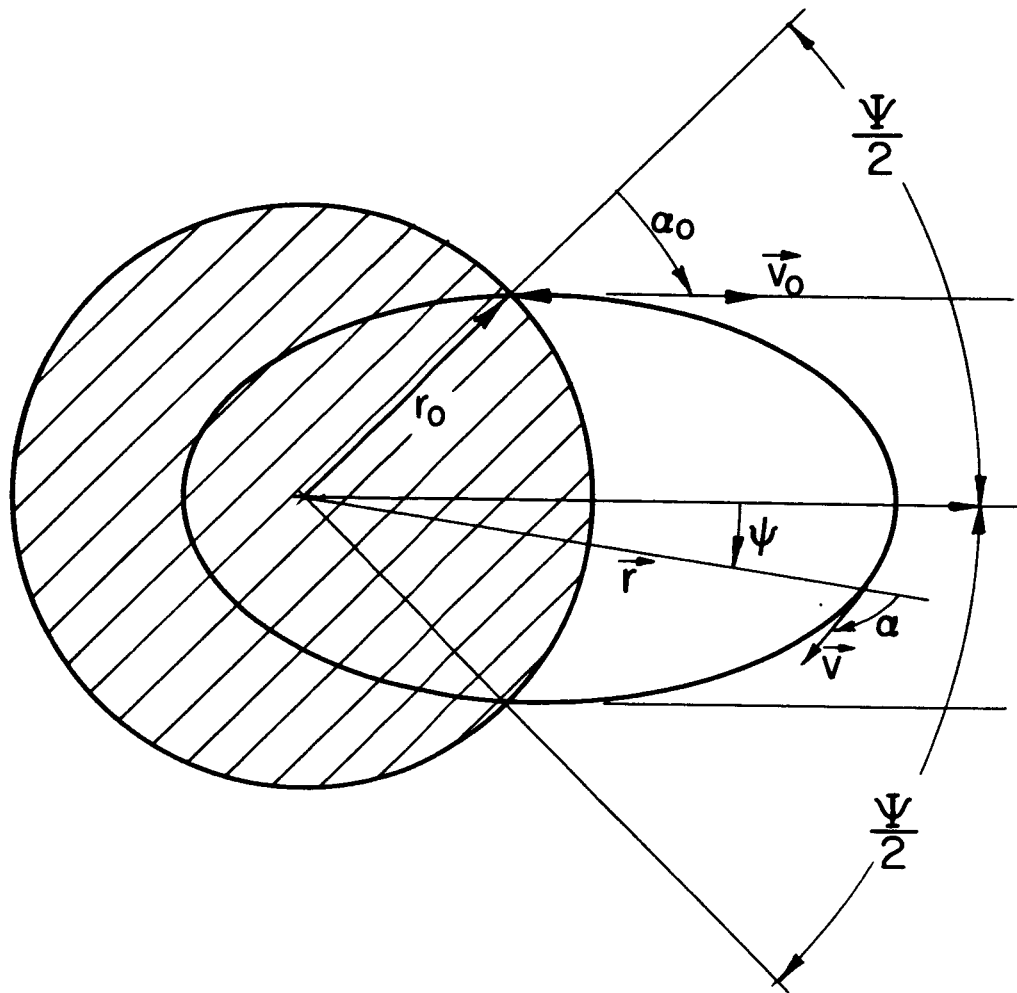


Fig. 2. Particle trajectory coordinates.

The law of motion can therefore be written

$$\frac{1}{r} = A - B \cos \psi, \quad (3-16)$$

where

$$A = \frac{2}{r_0} (v_{\text{orb}}/\bar{c})^2$$

and

$$B = \frac{2}{r_0} \sqrt{\left(\frac{v_{\text{orb}}}{\bar{c}}\right)^4 + \frac{1}{2} - \left(\frac{v_{\text{orb}}}{\bar{c}}\right)^2},$$

for $\alpha_0 = 45^\circ$. Here, $v_{\text{orb}} \equiv \sqrt{MG/r_0}$ is the orbital velocity at the surface; its value is 1.68 km/sec. (This is the velocity for which a circular orbit is possible just outside the lunar surface.) When $r = r_0$, $\psi = \frac{1}{2} \psi$, where ψ is the angular range in radians. The angular range is therefore given by

$$\cos \frac{\psi}{2} = \frac{A - \frac{1}{r_0}}{B} = \frac{\left(\frac{v_{\text{orb}}}{\bar{c}}\right)^2 - \frac{1}{2}}{\sqrt{\left(\frac{v_{\text{orb}}}{\bar{c}}\right)^4 + \frac{1}{2} - \left(\frac{v_{\text{orb}}}{\bar{c}}\right)^2}}. \quad (3-17)$$

The range is then given by

$$\rho = r_0 \psi = 2r_0 \arccos \left\{ \frac{1 - \frac{1}{2} \left(\frac{\bar{c}}{v_{\text{orb}}}\right)^2}{\sqrt{1 + \frac{1}{2} \left(\frac{\bar{c}}{v_{\text{orb}}}\right)^4 - \left(\frac{\bar{c}}{v_{\text{orb}}}\right)^2}} \right\}, \quad (3-18)$$

assuming $\bar{c} < v_{\text{esc}}$, the escape velocity. ($v_{\text{esc}} = \sqrt{2} v_{\text{orb}} = 2.38$ km/sec.) For the heavier atoms, such as argon, the ratio \bar{c}/v_{orb} may be small compared to 1 (for argon at 300°K, it is equal to 0.24.) The asymptotic form for the above equation for $\bar{c}/v_{\text{orb}} \ll 1$ is found as follows:

$$\rho = 2r_0 \arccos \left\{ \frac{1}{\sqrt{1 + \frac{\frac{1}{4} \left(\frac{\bar{c}}{v_{orb}} \right)^4}{\left[1 - \frac{1}{2} \left(\frac{\bar{c}}{v_{orb}} \right)^2 \right]^2}}} \right\}$$

$$\simeq 2r_0 \arccos \left\{ 1 - \frac{1}{8} \left(\frac{\bar{c}}{v_{orb}} \right)^4 \right\}.$$

Since $\cos x \simeq 1 - \frac{1}{2} x^2$, for $x \ll 1$,

$$\rho \simeq 2r_0 \left(\frac{\bar{c}}{\sqrt{2} v_{orb}} \right)^2 = r_0 \left(\frac{\bar{c}}{v_{orb}} \right)^2 = \frac{\bar{c}^2}{g}, \quad (3-19)$$

where g is the acceleration of gravity at the lunar surface, $g = MG/r_0^2$. This is just the expression which is found by neglecting the curvature of the surface and the variation of the acceleration of gravity, by solving the equation of motion

$$x = (\bar{c} \sin \alpha_0) t$$

$$z = (\bar{c} \cos \alpha_0) t - \frac{1}{2} g t^2 \quad (3-20)$$

for the case $\alpha_0 = 45^\circ$. The magnitude of the angular momentum constant is given by $P = m r^2 (d\psi/dt)$ for any time t . Therefore, the time of flight is given by

$$\begin{aligned}
t_f &= \frac{m}{P} \int_{-\psi/2}^{\psi/2} r^2 d\psi = \frac{2m}{P} \int_0^{\psi/2} \frac{d\psi}{(A-B \cos \psi)^2} \\
&= \frac{2m}{P} \left\{ \frac{B \sin \frac{\psi}{2}}{(A^2-B^2)(A-B \cos \frac{\psi}{2})} + \frac{2A}{(A^2-B^2)^{3/2}} \arctan \left[\sqrt{\frac{A+B}{A-B}} \tan \frac{\psi}{4} \right] \right\} \\
&= \frac{\sqrt{2} r_o}{\bar{c}} \left\{ \frac{1/2}{\left(\frac{v_{orb}}{\bar{c}}\right)^2 - \frac{1}{2}} \right. \\
&\quad \left. + \frac{\left(\frac{v_{orb}}{\bar{c}}\right)^2}{\left[\left(\frac{v_{orb}}{\bar{c}}\right)^2 - \frac{1}{2}\right]^{3/2}} \arctan \left[\frac{\left(\frac{v_{orb}}{\bar{c}}\right)^2 \sin \frac{\psi}{2} + \frac{1}{2}}{\left(\frac{v_{orb}}{\bar{c}}\right)^2 \sin \frac{\psi}{2} - \frac{1}{2}} \tan \frac{\psi}{4} \right] \right\} \quad (3-21)
\end{aligned}$$

If \bar{c}/v_{orb} is small compared to 1, the time of flight may be shown to have an asymptotic form as follows:

$$\begin{aligned}
t_f &= \frac{\sqrt{2} \bar{c} r_o}{v_{orb}^2} \left\{ \frac{1/2}{1 - \frac{1}{2} \left(\frac{\bar{c}}{v_{orb}}\right)^2} \right. \\
&\quad \left. + \frac{\frac{v_{orb}}{\bar{c}}}{\left[1 - \frac{1}{2} \left(\frac{\bar{c}}{v_{orb}}\right)^2\right]^{3/2}} \arctan \left[\frac{\sqrt{\left(\frac{v_{orb}}{\bar{c}}\right)^2 + \frac{1}{2 \sin \frac{\psi}{2}}} \cdot \frac{\sin \frac{\psi}{2}}{1 + \cos \frac{\psi}{2}} \right] \right\} \\
&\approx \frac{\sqrt{2} \bar{c} r_o}{v_{orb}^2} \left\{ \frac{1}{2} + \frac{v_{orb}}{\bar{c}} \arctan \left[\frac{1}{2} \left(\frac{\bar{c}}{v_{orb}}\right) \right] \right\}.
\end{aligned}$$

Since $\arctan x \approx x$, for $x \ll 1$,

$$t_f \approx \frac{\sqrt{2} \bar{c} r_o}{v_{orb}^2} = \frac{\sqrt{2} \bar{c}}{g} \quad (3-22)$$

This is just the expression found by neglecting the curvature of the surface and the variation of the acceleration of gravity.

For temperatures less than 267°K, the mean velocity for hydrogen is less than the escape velocity, and formulas (3-18) and (3-21) are applicable. (These formulas give an angular range 180° and an infinite time of flight, for hydrogen at T = 267°K, when the mean velocity is just equal to the escape velocity.) In Fig. 3, we have plotted the range and angular range for hydrogen and helium, as functions of the temperature of the surface where the flight begins, along with the asymptotic forms. For water vapor only the asymptotic form is shown. Typical values of range and time of flight are given in Table 1. (The times of flight have been calculated using the asymptotic formula except for helium and hydrogen; we have used the exact formula for helium.) Figure 3 shows that the range versus temperature curve is much steeper for hydrogen than for the other gases. This means that migration of hydrogen atoms will be much more efficient from the hot side to the cold side than in the opposite direction. Therefore, even though hydrogen is supplied only on the sunlit side, the density at the surface is likely to be fairly uniform over the entire surface of the moon.

TABLE 1

ATMOSPHERIC PARTICLE RANGE, TIME OF FLIGHT AND SCALE HEIGHT
(T = 300°K)

Gas	Mean Thermal Speed, \bar{c} (cm/sec)	Range ρ and Angular Range ψ ($\alpha = 45^\circ$) (km)		Time of Flight, t_f (sec)	Scale Height, h (km)	$\frac{\pi r_0^2}{V}$ (cm ⁻¹)
		(T = 300°K)	(T = 130°K)			
H	2.51×10^5 *		2632 (87°)		1530**	7.9×10^{-10}
He	1.26×10^5	1316 (43°)	483 (16°)	1923	385	5.3×10^{-9}
H ₂ O	5.94×10^4	217 (7°)	93.6 (3°)	519	85.6	2.8×10^{-8}
Ar	3.98×10^4	97.9 (3.2°)	42.3 (1.4°)	348	38.6	7×10^{-8}
Kr	2.75×10^4	46.5 (1.5°)	20.1 (0.7°)	240	18.4	1.5×10^{-7}
Xe	2.19×10^4	29.8 (1°)	12.8 (0.4°)	191	11.7	2.3×10^{-7}

*Compare this with the escape velocity, $v_{esc} = 2.37 \times 10^5$ cm/sec.

**Compare this with the lunar radius, $r_0 = 1740$ km.

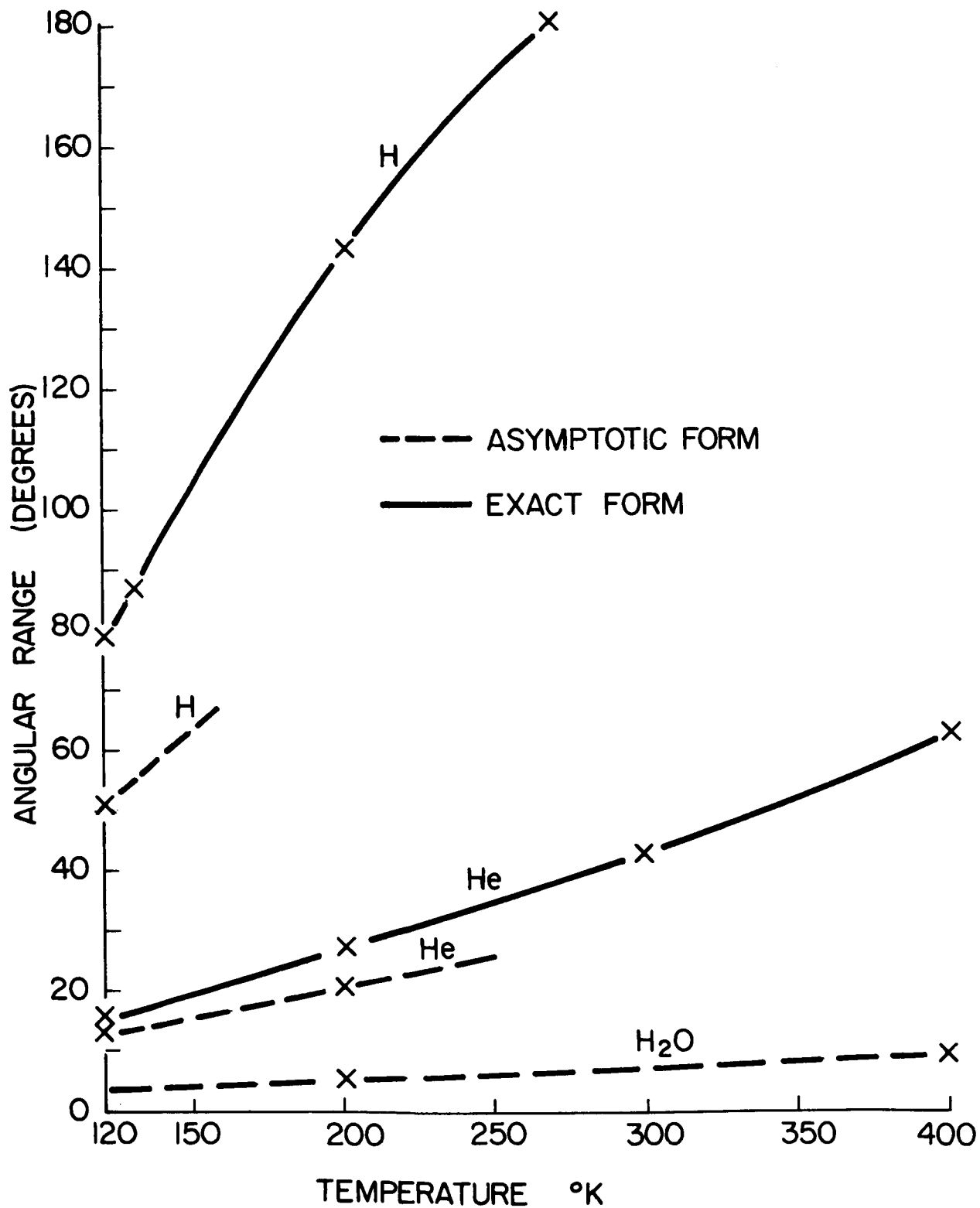


Fig. 3. Angular range vs. temperature.

C. THE BAROMETRIC LAW

If no collisions take place between particles in the atmosphere, the distribution function is a solution of the partial differential equation

$$\frac{\partial f}{\partial t} + \sum_{s=1}^3 \left[\frac{\partial f}{\partial q_s} \dot{q}_s + \frac{\partial f}{\partial p_s} \dot{p}_s \right] = 0 ,$$

as we have shown in Section III-A. The assumption of no collisions will be justified later on in this section, and we will use it for now without justification.

Let us assume that the surface of the moon is at the constant temperature T and that the atmosphere is in equilibrium out to large distances, that is, just as much mass, momentum, and energy is being transported outward as inward at any given time and position. Then we can consider the particles to start or end their flights, or both, at the lunar surface. In this case, the function f can be found if it is given at the surface; we assume of course, that f does not depend upon the time explicitly: $\partial f / \partial t = 0$. Let us assume that f takes the following form at $r = r_0$:

$$f_0 \equiv f(r_0, \theta, \phi, p_r, p_\theta, p_\phi) = n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-p^2/2mkT}, \quad (3-23)$$

where

$$p^2 = p_r^2 + \frac{1}{r^2} \left(p_\theta^2 + \frac{p_\phi^2}{\sin^2 \theta} \right) = m^2 (v_x^2 + v_y^2 + v_z^2),$$

and n_0 is the number density of the particles at the surface. This is simply n_0 multiplied by the Maxwellian velocity distribution formula for a gas in contact with a surface at temperature T . The only function of the kind $F(I_1, \dots, I_6)$ of Eq. (3-10) which takes the above form at $r = r_0$ is the following:

$$f(q_1, \dots, p_3) = n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mMG}{kTr_0}} e^{-\frac{1}{kT} H(q_1, \dots, p_3)}. \quad (3-24)$$

This is the molecular distribution function, sometimes called the Maxwell-Boltzmann distribution, for an atmosphere where the following assumptions are valid:

1. No collisions take place between particles in the atmosphere.

2. The entire lunar surface is at the same temperature T .
3. The atmosphere is in equilibrium out to large distances.

Although the assumptions valid for an isothermal atmosphere in equilibrium with no collisions are not strictly valid for the lunar atmosphere, as described in the next section, the results for this idealized atmosphere are extremely useful and instructive. If we integrate the Maxwell-Boltzmann distribution function over the entire momentum space, we get the spacial distribution of density in the isothermal atmosphere:

$$\begin{aligned}
 n(r, \theta, \phi) &= \iiint f dp_r dp_\theta dp_\phi \\
 &= n_0 \exp \left[\frac{mMG}{kT} \left(\frac{1}{r} - \frac{1}{r_0} \right) \right] \\
 &= n_0 \exp \left[\frac{r_0}{h} \left(\frac{r_0}{r} - 1 \right) \right] , \tag{3-25}
 \end{aligned}$$

where

$$h = \frac{kTr_0^2}{mMG} = \frac{kT}{mg} .$$

This is called the barometric law, and h is called the scale height. The scale height is a measure of the rate of decrease of density with increasing height. Scale heights for various gases are given for $T = 300^\circ\text{K}$ in Table 1. To help in visualizing the shape of the atmospheric distribution of density, we plot, in Fig. 4, the scale height for hydrogen, helium, and water vapor at the lunar equator, as functions of the angle which the sun makes with the zenith.

1. Gravitational Condensation

We may now use the barometric law to calculate the effect of gravitational condensation of interplanetary material: the increase in density of the interplanetary material near the moon. If this material were in hydrostatic equilibrium at the temperature T , then the ratio of the density at the lunar surface to the density far away from the moon is given by

$$\frac{n_0}{n_\infty} = e^{r_0/h} , \tag{3-26}$$

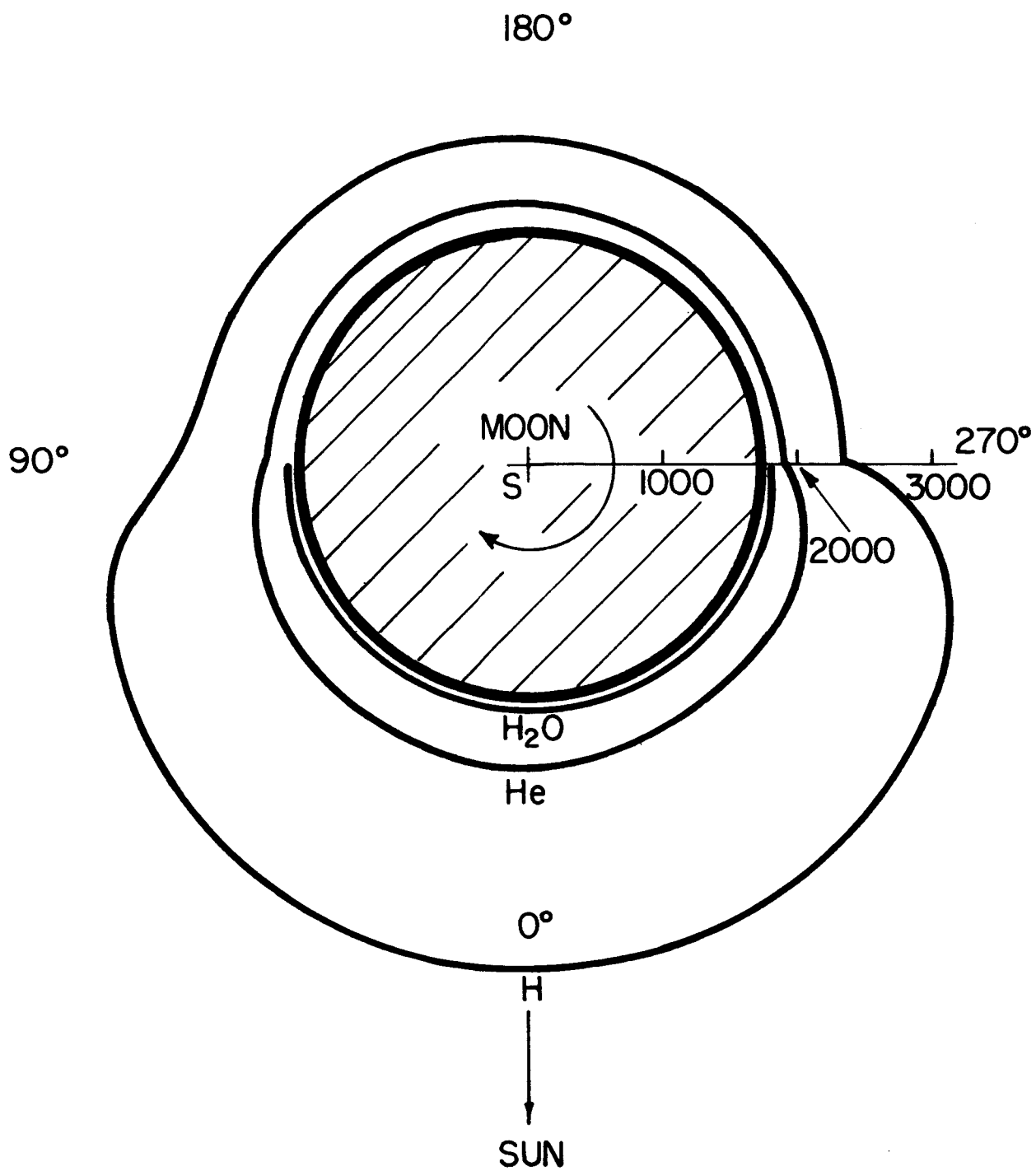


Fig. 4. Scale height vs. sun angle; dimensioned from center of moon.

where

$$h = \frac{kTr_0^2}{mMG} ,$$

and m is the mean weight of particles in the interplanetary material. If we take the interplanetary material to be fully ionized with a mean molecular weight of about $1/2$ and a temperature of about 10^5 °K, then the above ratio is

$$\frac{n_0}{n_\infty} = 1.0002 .$$

This is obviously too small to explain Elsmore's result for the increased electron density near the moon. The gravitational condensation of interplanetary gas has been discussed by Firsoff,²¹ Brandt,²² and Singer.²³

2. The Scale Volume

If we introduce the height above the lunar surface, $z = r - r_0$, the barometric law may be approximated as follows, for $z \ll r_0$:

$$n = n_0 \exp \left[\frac{r_0}{h} \left(\frac{r_0}{r_0 + z} - 1 \right) \right] \simeq n_0 e^{-z/h} . \quad (3-27)$$

If the last expression were valid for all z , we could integrate to get "the total number of particles in a square centimeter column":

$$N_c = \int_0^\infty n_0 e^{-z/h} dz = n_0 h . \quad (3-28)$$

Although this number does not exist, since the number density does not really go to zero with increasing height above the surface, it may give a good estimate for the number of particles in the unit column which are associated with the atmosphere, especially if the scale height is small in comparison to the lunar radius: $h \ll r_0$. This is further substantiated by the work of Herring and Kyle.²⁴ They have derived the distribution of density in an exosphere without assuming that it is in equilibrium out to large distances. We will give their result in Eq. (3-46); they have shown by integrating this equation numerically that the number of particles in a square centimeter column which began their trajectories at the lunar surface is close to $n_0 h$ for large values of r_0/h , although it departs considerably for smaller values of r_0/h than about 40.

The total number of particles which are associated with the atmosphere is then gotten by multiplying by the surface area of the moon:

$$N = 4\pi r_0^2 h n_0 . \quad (3-29)$$

If $h \ll r_0$, this is approximately equal to the density at the surface multiplied by the scale volume V , the volume contained between the surface and a sphere at $r = r_0 + h$. We can then write this fictitious number as $N = n_0 V$, where V , the scale volume is given by

$$V = \frac{4}{3} \pi [(r_0 + h)^3 - r_0^3] . \quad (3-30)$$

If $h \ll r_0$, $V \simeq 4\pi r_0^2 h$, and N is given by Eq. (3-29) above.

3. The Assumption of No Collisions

Finally, we wish to justify the assumption of no collisions in the lunar atmosphere. The so-called Maxwell mean-free-path formula is

$$\lambda = \frac{1}{4\sqrt{2} \pi a^2 n_0} , \quad (3-31)$$

where πa^2 is the gas kinetic cross section of the particles. As a criterion for a collisionless atmosphere, let us set the mean-free-path equal to the scale height:

$$\frac{1}{4\sqrt{2} \pi a^2 n_0} = \frac{kT}{mg} . \quad (3-32)$$

The upper limit for the density at the surface for a collisionless atmosphere is then given by

$$n_0 = \frac{mg}{4\sqrt{2} \pi a^2 kT} . \quad (3-33)$$

The smallest value of n_0 is given by this formula in the case of hydrogen. At $T = 300^\circ\text{K}$, Eq. (3-33) gives $n_0 = 2.5 \times 10^6$ atoms/cm³. Elsmore's estimate for the upper limit of the density of neutral particles is of this order of magnitude. Therefore, the mean-free-path for particles in the lunar atmosphere is likely to be long enough to allow us to neglect collisions.

Another way of justifying this is to compare the time of flight with the mean-free-time. Using the Maxwell mean-free-path formula, the mean-free-time is given by

$$\tau = \frac{1}{4 \sqrt{2} \pi a^2 n_0 \bar{c}} \quad (3-34)$$

For helium at 300°K, at a density of $10^6/\text{cm}^3$, the mean-free-time is $\tau = 3762$ sec. The time of flight is, from Table 1, 1923 sec. The mean-free-time is calculated for a flight in a region of constant density, and since it is, even so, about twice as great as the time of flight itself, we see that collisions are unlikely.

D. CRITICISM OF THE USE OF THE BAROMETRIC LAW FOR THE LUNAR ATMOSPHERE

The following are the reasons why the assumptions valid for an isothermal atmosphere in equilibrium with no collisions are not strictly valid for the lunar atmosphere.

1. Collisions will always take place, even though they are extremely infrequent. These will put some particles into bound orbits around the moon which do not intersect its surface. The number of these orbiting particles is difficult to calculate, and the motions of these particles are not described by giving f at the surface.

2. The temperature of the lunar surface varies from about 400°K at the subsolar point to about 120°K at the coldest parts of the lunar surface. Therefore, the function f cannot be given by the same expression over the entire surface. Also, because of the long range of the atoms of hydrogen and helium, these particles will always be arriving at certain places on the lunar surface with velocities characteristic of the temperature at certain other places on the lunar surface. The distribution function at any given place on the lunar surface is very difficult to find because of this situation.

3. The atmosphere is not in equilibrium out to large distances; this state would require an infinite length of time to establish. Because of this, particles which leave the surface of the moon with velocities exceeding the escape velocity will be lost from the atmosphere, since they suffer no collisions, and these will not be compensated for by particles coming from a large distance, with correspondingly high velocities. The distribution of velocities will therefore be truncated for velocities directed toward the moon and exceeding the escape velocity. (Also, the atmosphere will eventually escape entirely, unless some supply mechanism exists on the lunar surface, because of the escape, or evaporation, of the tail of the velocity distribution. The rate of escape of the lunar atmosphere due to this mechanism is calculated in Section III-E.)

Let us now find the distribution of density in the lunar atmosphere assuming only that no collisions take place and that the temperature of the moon is constant over the surface. We will no longer assume that the atmosphere is in equilibrium out to large distances. In this we use Liouville's theorem, which was derived in Section III-A, which states that the density of representative points in phase space, f (the molecular distribution function) is a constant along a dynamical trajectory. It will be convenient to express f as a function of position and velocity:

$$f(r, \theta, \phi, v_r, v_\theta, v_\phi) ,$$

rather than as a function of position and momentum, as we have done previously. The relations which show how the velocities replace the momenta are as follows:

$$p_r = mv_r, \quad p_\theta = mr v_\theta, \quad p_\phi = mr \sin \theta v_\phi .$$

Liouville's theorem can then be expressed by the equation

$$f(r, \theta, \phi, v_r, v_\theta, v_\phi) = f_0(r_0, \theta_0, \phi_0, v_{r0}, v_{\theta0}, v_{\phi0}) , \quad (3-35)$$

where f_0 is the function f evaluated at the point on the surface where the trajectory began. The variables in the two functions above are related as follows, using the conservation of energy and angular momentum:

$$\begin{aligned} v^2 - \frac{2MG}{r} &= v_0^2 - \frac{2MG}{r_0} \\ vr \sin \alpha &= v_0 r_0 \sin \alpha_0 . \end{aligned} \quad (3-36)$$

Here, v is the magnitude of the velocity, α is the angle between the velocity vector and the radius vector \vec{r} , and subscripts 0 indicate the values of these variables at the point on the lunar surface where the trajectory began. Assuming the density at the lunar surface does not depend upon θ or ϕ , the spacial distribution of density in the atmosphere is given by

$$n(r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(r, v_r, v_\theta, v_\phi) dv_r dv_\theta dv_\phi . \quad (3-37)$$

Using spherical coordinates, $v_r = v \cos \alpha$, $v_\theta = v \sin \alpha \cos \beta$, $v_\phi = v \sin \alpha \sin \beta$, the integral is

$$n(r) = \int_0^{2\pi} \int_0^\pi \int_0^\infty f(r, v, \alpha, \beta) v^2 \sin \alpha \, dv d\alpha d\beta. \quad (3-38)$$

Using Liouville's theorem, this may be written as follows:

$$n(r) = \int_0^{2\pi} \int_0^\pi \int_0^\infty f_0(r_0, v_0, \alpha_0, \beta_0) v^2 \sin \alpha \, dv d\alpha d\beta. \quad (3-39)$$

The density can be found if we know the molecular distribution function at the surface, assuming that each particle started its trajectory at the surface. (The density of particles in bound elliptic orbits which do not intersect the surface of the moon cannot be found in this way.)

To carry out the integration, we need to transform the volume element $dv d\alpha d\beta$ to one involving v_0 , α_0 , and β_0 . Using the relations in Eq. (3-36) and the fact that $\beta = \beta_0$, we find that the functional determinant has the value

$$\frac{\partial(v, \alpha, \beta)}{\partial(v_0, \alpha_0, \beta_0)} = \left(\frac{v_0}{v}\right)^2 \frac{r_0}{r} \frac{\cos \alpha_0}{\cos \alpha}. \quad (3-40)$$

The integral can then be written as follows:

$$n(r) = \int_0^{2\pi} \int_0^\pi \int_{v_{\min}}^\infty f_0(r_0, v_0, \alpha_0, \beta_0) v_0^2 \frac{r_0}{r} \frac{\sin \alpha}{\cos \alpha} \cos \alpha_0 \, dv_0 d\alpha_0 d\beta_0. \quad (3-41)$$

Since the range of integration must be over the part of velocity space compatible with the relations in Eq. (3-36), i.e., only over those orbits which intersect the sphere at r_0 and r , the lower limit in the integration over v_0 is the minimum velocity for which a particle may reach the sphere at r if it is emitted from r_0 at the angle α_0 . From the relations given in Eq. (3-36), we find

$$v_{\min}^2 = \frac{2MG\left(\frac{1}{r_0} - \frac{1}{r}\right)}{1 - \left(\frac{r_0}{r}\right)^2 \sin^2 \alpha_0}. \quad (3-42)$$

To express the integrand entirely in terms of v_0 , α_0 , and β_0 , we again use the relations in Eq. (3-36). We have

$$\frac{\sin \alpha}{\cos \alpha} = \frac{\frac{r_0}{r} \sin \alpha_0}{\sqrt{1 + \frac{2MG}{v_0^2} \left(\frac{1}{r} - \frac{1}{r_0} \right) - \left(\frac{r_0}{r} \right)^2 \sin^2 \alpha_0}} \quad (3-43)$$

Therefore, the integral becomes

$$n(r) = \int_0^{2\pi} \int_0^\pi \int_{v_{\min}}^\infty f_0(r_0, v_0, \alpha_0, \beta_0) \left(\frac{r_0}{r} \right)^2 \frac{v_0^2 \cos \alpha_0 \sin \alpha_0 dv_0 d\alpha_0 d\beta_0}{\sqrt{1 + \frac{2MG}{v_0^2} \left(\frac{1}{r} - \frac{1}{r_0} \right) - \left(\frac{r_0}{r} \right)^2 \sin^2 \alpha_0}} \quad (3-44)$$

Now we use the truncated Maxwellian distribution for f_0 :

$$f_0 = \left\{ \begin{array}{ll} n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv_0^2}{2kT}}, & \left\{ \begin{array}{l} 0 < v_0 < \infty \\ 0 < \alpha < \pi/2 \end{array} \right\} \text{ and } \left\{ \begin{array}{l} 0 < v_0 < v_{\text{esc}} \\ \pi/2 < \alpha < \pi \end{array} \right\} \\ 0, & \left\{ \begin{array}{l} v_{\text{esc}} < v_0 < \infty \\ \pi/2 < \alpha < \pi \end{array} \right\} \end{array} \right\} \quad (3-45)$$

(Note that the factors v_0^2 and $\sin \alpha$ have already been included, in the functional determinants.) The integral can now be written as follows:

$$\begin{aligned} n(r) = & 2\pi \left(\frac{r_0}{r} \right)^2 \int_0^\pi \int_{v_{\min}}^{v_{\text{esc}}} n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv_0^2}{2kT}} \frac{v_0^2 \cos \alpha_0 \sin \alpha_0 dv_0 d\alpha_0}{\sqrt{1 + \frac{2MG}{v_0^2} \left(\frac{1}{r} - \frac{1}{r_0} \right) - \left(\frac{r_0}{r} \right)^2 \sin^2 \alpha_0}} \\ & + 2\pi \left(\frac{r_0}{r} \right)^2 \int_0^{\pi/2} \int_{v_{\text{esc}}}^\infty n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv_0^2}{2kT}} \frac{v_0^2 \cos \alpha_0 \sin \alpha_0 dv_0 d\alpha_0}{\sqrt{1 + \frac{2MG}{v_0^2} \left(\frac{1}{r} - \frac{1}{r_0} \right) - \left(\frac{r_0}{r} \right)^2 \sin^2 \alpha_0}} \quad (3-46) \end{aligned}$$

This result has been obtained by Öpik and Singer²⁵ and by Herring and Kyle.²⁴ The latter authors have evaluated the integral as follows:

$$\begin{aligned}
\frac{n(r)}{n_0} = & \left\{ e^{-\frac{r_0}{h}(1 - \frac{r_0}{r})} \left(1 - \frac{1}{2} \operatorname{erfc} \frac{r_0}{\sqrt{hr}} \right) \right. \\
& - \sqrt{1 - \left(\frac{r_0}{r} \right)^2} e^{-r_0/h(1 + \frac{r_0}{r})} \left(1 - \frac{1}{2} \operatorname{erfc} \frac{r_0}{\sqrt{h(r+r_0)}} \right) \\
& \left. - \frac{r_0}{\sqrt{\pi hr}} \left(1 - \sqrt{1 - \frac{r_0}{r}} \right) e^{-r_0/h} \right\}. \quad (3-47)
\end{aligned}$$

For values of r/r_0 close to one, the above equation departs very little from the barometric law, particularly in the case of a heavy gas such as argon. For larger values of r/r_0 , the above equation gives values of $n(r)/n_0$ considerably smaller than the barometric law, and the ratio $n(r)/n_0$ goes to zero as r/r_0 gets very large. This is a more realistic result than that given by the barometric law, which predicts the constant value $e^{-r_0/h}$ for $n(r)/n_0$ as r/r_0 becomes very large.

E. THE JEANS EVAPORATION FLUX

In Section III-D we mentioned the fact that some high velocity particles will escape from the atmosphere and will not be compensated for by incoming particles. This evaporation of the tail of the velocity distribution was first discussed by Jeans.²⁶ Since the escape velocity at the surface is

$$v_{\text{esc}} = \sqrt{2MG/r_0},$$

the number of particles which leave one cm^2 of the surface/sec with velocities exceeding the escape velocity is

$$J_J = \iiint f_0 v_z dv_x dv_y dv_z. \quad (3-48)$$

$$(v_z > 0, v_x^2 + v_y^2 + v_z^2 > \frac{2MG}{r_0})$$

$$\begin{aligned}
J_J &= n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} \pi \int_{\sqrt{2MG/r_0}}^{\infty} e^{-mc^2/2kT} c^3 dc \\
&= \frac{n_0 \bar{c}}{4} e^{-r_0/h} \left(1 + \frac{r_0}{h} \right). \quad (3-49)
\end{aligned}$$

The Jean's evaporation flux is seen to be a sensitive function of the quantity

$$\frac{r_0}{h} = \frac{r_0 mg}{kT}.$$

The smaller the mass of the atoms and the higher their temperature, the more readily they escape by this process. In order for the density of any gas in the lunar atmosphere to be constant over long periods of time, some supply must exist to balance the escape of gases by this and other processes.

F. SOURCES FOR THE LUNAR ATMOSPHERE

It will be seen in Section IV that an atmosphere cannot exist on the moon without having a continuous gas supply from some source. In this report we will consider the following sources.

(a) The Solar Wind, as a Source of Neutral Hydrogen and Helium.—We assume that every positively charged particle from the solar wind which strikes the moon's surface is confined within the crystal lattice of the surface material for a certain length of time and then re-emitted as a neutral atom, such that the velocity distribution of the emitted particles is Maxwellian at the temperature of the surface where they are emitted. (The velocity distribution we are speaking of has only velocities directed away from the surface, so it is not Maxwellian in the strict sense.) We assume that the number of particles does not buildup in the surface layer, so that the incoming flux of charged particles equals the emitted flux of neutrals, and we call this process "diffuse reflection." (Actually, a certain number of the particles will be specularly reflected and not accommodated to the surface temperature, and possibly not neutralized, but for simplicity we neglect these.)

(b) The Supply of H₂O, CO₂, and SO₂ From the Crustal Rocks, From Hot Springs, From Lava Flows and From Volcanos, and the Supply of H₂O From Trapped Ice.—Assuming that the composition of the moon is similar to the earth's crust and mantle, and that the yearly yield of volatiles is proportional to surface area, Vestine²⁷ predicts equivalent source strengths for these gases on the moon as follows:

H₂O: 4.5×10^{10} molecules/cm² sec
CO₂: 10^{10} molecules/cm² sec
SO₂: 1.6×10^8 molecules/cm² sec.

It is unlikely that the actual source strength for water vapor is as high as that given above. If appreciable amounts of hydrated silicate were present in the material that formed the moon, it is possible that water could be trapped below the surface as permafrost. Since the temperature at shallow depths is so low that the evaporation rate would be very small, all of the water contained in the moon could not escape readily into the atmosphere. The possibility of permafrost on the moon has been discussed by Gold.²⁸

Watson, Murray, and Brown²⁹ have discussed the possibility of ice being retained on the moon on portions of its surface which are permanently shaded from the sun. If these portions of the surface are at a temperature of about 120°K, the estimated evaporation rate is 10^{-15} grams/cm² sec, which is about 3×10^7 molecules/cm² sec. These authors estimate that the fraction of the lunar surface that is permanently shaded is most likely about 0.005. The equivalent source strength for water vapor based upon the entire lunar surface is therefore about 1.5×10^5 molecules/cm² sec.

Although the estimated source strengths for CO₂ and SO₂ are high enough that these gases may constitute an appreciable portion of the lunar atmosphere, they will be ignored here because of the impossibility of treating them with the simple model of Section IV, where photochemical reactions and chemical combination on the lunar surface are not considered. It must be stressed, however, that our simple model will give only a minimum estimate for the lunar atmospheric density, because we have not considered these gases.

(c) Natural Radioactivity of K⁴⁰ as a Source of Ar.—Assuming that about 0.12% by weight of the moon is composed of potassium, of which about 0.011% is K⁴⁰, and that each gram of K⁴⁰ has produced 0.1 gram of Ar by electron capture or positron decay processes, as has apparently been the case for the earth, then Vestine²⁷ calculates that the number of atoms of Ar produced/cm² of the lunar surface over a period of 3×10^9 years would be 4.0×10^{22} atoms of Ar/cm².

The half-life of K⁴⁰ is about 1.4×10^9 years, and the activity is not constant over any period of time comparable with the half-life. If $a(t)$ is the activity in disintegrations/sec, per cm² of surface, then

$$a(t) = a_0 2^{-t/t_{1/2}}, \quad (3-50)$$

where a_0 is the original activity and $t_{1/2}$ is the half-life. The number of atoms of Ar produced/cm² of surface in a time t is

$$N(t) = \int_0^t a(t) dt = \frac{a_0 t_{1/2}}{\ln 2} (1 - 2^{-t/t_{1/2}}). \quad (3-51)$$

Therefore, the activity at time t is

$$a(t) = \frac{N(\ln 2)}{t_{1/2}(2^{t/t_{1/2}} - 1)} \quad (3-52)$$

Using $N = 4.0 \times 10^{22}/\text{cm}^2$ and $t = 2t_{1/2}$, we find $a = 1.9 \times 10^5/\text{cm}^2 \text{ sec}$. Assuming that the time of outgassing for the Ar atoms is much smaller than the half-life, the source strength of Ar at the surface is the same as the activity, 1.9×10^5 atoms of Ar/cm² sec.

(d) Production of Krypton and Xenon by Spontaneous Fission of U^{238} , Decay of I^{129} , and From Primeval Gases Trapped in Rocks and Released by Meteor Bombardment.—These processes have been examined by Edwards and Borst.³⁰ The primeval gases give the largest contribution; in a layer 10^4 cm deep they give the following values for the total number of atoms:

Kr: 10^{33} atoms

Xe: 7×10^{31} atoms

If we assume that these are released at a constant rate over the entire surface by meteor excavation over a period of 3×10^9 years, the equivalent source strengths are:

Kr: 2.8×10^{-2} atoms/cm² sec

Xe: 2×10^{-3} atoms/cm² sec.

TABLE 2

ASSUMED SOURCE STRENGTHS

Gas	Source Strength (atoms/cm ² sec)	Area of Source
H	.93 J	πr_0^2
He	$7 \times 10^{-2} J$	πr_0^2
H ₂ O	1.5×10^5	$4\pi r_0^2$
Ar	1.9×10^5	$4\pi r_0^2$
Kr	2.8×10^{-2}	$4\pi r_0^2$
Xe	2×10^{-3}	$4\pi r_0^2$

G. THE EFFECT OF THE SOLAR WIND ON THE LUNAR ATMOSPHERE

1. Assumptions Concerning the Solar Wind

To calculate the effect of the solar wind on the lunar atmosphere, we must make some specific assumption about the parameters of interest. Since the solar wind flux, J , is known to vary with solar activity, but its magnitude is not well known for any specific level of solar activity, the densities of atmospheric gases predicted by our model in Section IV will be presented as a function of J , for values of J between $10^7/\text{cm}^2 \text{ sec}$ and $10^{13}/\text{cm}^2 \text{ sec}$.

We must, however, be more specific for assumptions concerning the velocity of the solar wind so that we may predict the elastic scattering and charge exchange cross sections for the various atmospheric gases. For the mean flow velocity of the solar stream, we will use the value $\langle u \rangle_{\text{avg}} = 500 \text{ km/sec}$, which is, according to Parker,⁶ the minimum velocity for a quiet sun. This assumption is somewhat incompatible with our use of a variable flux; however, we are primarily interested in conditions during quiet sun, and the problem is much less complicated if we make the assumption that $\langle u \rangle_{\text{avg}}$ is constant.

To exemplify the use of the equations in Section IV, we will use the value $J = 10^9/\text{cm}^2 \text{ sec}$. Since this is the highest flux measured by the Russians during an average geomagnetic storm, we would be persuaded to assume an average value of J for quiet solar conditions less than this. However, the majority of the remaining theoretical evidence is for higher values of J , even during quiet solar conditions. More recent measurements by U. S. satellites corroborate the Russian measurements, so that a flux of about $10^9 \text{ part/cm}^2 \text{ sec}$ is a reasonable value to assume.

The above two assumptions for J and $\langle u \rangle_{\text{avg}}$ force us to use the value $n = 20/\text{cm}^3$ for the number of ion-electron pairs/ cm^3 in the solar wind. Hinteregger³¹ has shown that the ratio of densities of neutral and ionized particles in the interplanetary space is unlikely to exceed 10^{-3} . Because this is the result of considering only photoionization, we expect this ratio to be even lower when the ionizing effects of the solar wind are taken into consideration. Therefore, we will neglect the presence of any neutral particles in the solar wind and assume that it is completely ionized.

The question now arises concerning the temperature of the interplanetary gas, or the solar wind. Pottasch³² has computed the temperature distribution from 1.0043 to 20 solar radii in the solar corona, using the observed distribution of density in this region and the assumption of hydrostatic equilibrium. If we extrapolate his curve to the distance of the earth's orbit, we find the value $T \approx 10^5 \text{ K}$. This is about equal to the temperature which was calculated by Chamberlain³³ using an exospheric model of the solar corona, and is somewhat less than the temperature found by Chapman³⁴ based upon heat conduction in a static, spherically symmetric model solar corona.

We will assume that the solar wind is simply the hydrodynamic expansion of the solar corona, so that helium is present as well as hydrogen. Assuming that the composition of the corona by weight is 75% hydrogen, 23% helium, and 2% heavy elements with a mean atomic weight of about 32, all completely ionized, the number density of helium ions should be about 6/75 of that of hydrogen ions or about 7% of the total density of positive ions in the stream. Since collision frequencies in the corona close to the sun can be assumed relatively high, we will assume that the helium ions travel at the same mean velocity in the solar stream as the hydrogen ions and the electrons.

Although the solar wind is believed to flow radially outward from the sun, we will assume that the flow in the vicinity of the earth is a one-dimensional uniform flow, over a region the size of the moon, in the direction away from the sun. The ordered mass motion of the solar stream at 500 km/sec corresponds to kinetic energies of 1300 electron volts for protons, 5200 ev for helium ions, and 0.7 ev for electrons. At the temperature $T = 10^5$ °K, the thermal energy per particle is about 15 ev for all kinds of particles, assuming that equipartition of energy exists. (This assumption will be discussed in the last part of this section.) We see that the energy of the ordered motion is much greater than the energy of the random motion for protons and helium ions, and is considerably less than the random energy for electrons. The physical situation is very similar to that of a spherical satellite moving with a uniform speed in the earth's ionosphere, with a velocity much greater than the thermal velocities of the positive ions but less than the root-mean-square thermal velocity of the electrons.

For the purpose of describing various physical processes, we will assume that the velocity distribution for the particles in the solar wind is Maxwellian and isotropic in a coordinate system moving with the mean velocity of the flow. This assumption may be partially justified by comparing the times of relaxation for the particles and the length of time necessary for the particles to travel from the sun to the orbit of the earth.

The "self-collision time" for positive ions in a plasma interacting with others of the same species is given by Spitzer³⁵ as follows:

$$t_c = \frac{11.4 A^{1/2} T^{3/2}}{n Z^4 \ln \Lambda}, \quad (3-53)$$

where

$$\Lambda = \frac{3}{2 Z^2 e^3} \left(\frac{k^3 T^3}{\pi m} \right)^{1/2} = \frac{1.24 \times 10^4}{Z^2} \sqrt{T^3/n},$$

A is the atomic weight, and Z is the number of unit charges of the positive ions. This is the time in which a particle is gradually deflected by 90°

due to the interactions with other particles, on the average. The self-collision time is a measure both of the time required to reduce substantially any lack of isotropy in the velocity distribution and also of the time required for the distribution of kinetic energies to approach the Maxwellian distribution. If $T = 10^5 \text{°K}$ and $n = 20/\text{cm}^3$, then $t_c = 7 \times 10^5 \text{ sec}$, for H.

This is of the same order of magnitude as the length of time taken for protons traveling at 500 km/sec in traveling from the sun. The self-collision time for electrons in the plasma is a fraction $\sqrt{m_e/m_p} = 1/43$ of that for protons, which is considerably smaller than the time taken in traveling from the sun. Therefore, the particles may be assumed to have velocity distributions which are Maxwellian and isotropic, as stated previously. The temperature may be different, however, for the different kinds of particles.

The rate at which equipartition is established between protons and electrons is measured by the "time of equipartition," $t_{eq} = 43t_c = 3 \times 10^7 \text{ sec}$. Thus, if the particle density was as low as $20/\text{cm}^3$ during the entire duration of time in which the stream travels from the sun, it is unlikely that the temperatures of electrons and positive ions would be the same at the earth's orbit if they were not the same in the solar corona. It is possible, however, that equipartition may have become established closer to the sun, where the density is higher. We will assume here that the temperatures of all kinds of particles in the solar wind at the orbit of the earth are the same, $T = 10^5 \text{°K}$.

Table 3 summarizes our assumptions concerning the solar wind. The number nE_{flow} is the number density multiplied by the kinetic energy per particle, or the kinetic energy density. The total for all three types of particles is about $5 \times 10^{-8} \text{ ergs/cm}^3$, as was mentioned in Section II-C.

TABLE 3

THE ASSUMED MINIMAL MODEL OF THE SOLAR WIND

	Electrons e^-	Protons H^+	Helium Ions He^{++}
J	$10^9/\text{cm}^2 \text{ sec}$	$0.93 \times 10^9/\text{cm}^2 \text{ sec}$	$7 \times 10^7/\text{cm}^2 \text{ sec}$
$\langle u \rangle_{\text{avg}}$	500 km/sec	500 km/sec	500 km/sec
E_{flow}	0.7 ev = $1.1 \times 10^{-12} \text{ erg}$	1300 ev = $2.1 \times 10^{-9} \text{ erg}$	5200 ev = $8.3 \times 10^{-9} \text{ erg}$
n	$20/\text{cm}^3$	$18.6/\text{cm}^3$	$1.4/\text{cm}^3$
nE_{flow}	$2.16 \times 10^{-11} \text{ ergs/cm}^3$	$3.87 \times 10^{-8} \text{ ergs/cm}^3$	$1.6 \times 10^{-8} \text{ ergs/cm}^3$
T	10^5°K	10^5°K	10^5°K
$\frac{3}{2} kT$	15 ev	15 ev	15 ev

2. Elastic Scattering

The molecules in the lunar atmosphere will be subjected to collisions with the high energy protons. The difference between protons and helium ions will be ignored here and we will also ignore the effect of the electrons, since their energy is so low. These collisions may ionize an atmospheric molecule, effect a charge-exchange reaction, or collide elastically, with no change in the internal states of the colliding particles. As shown by Massey and Burhop,³⁶ charge exchange is about an order of magnitude more probable than ionization, so it is the only inelastic collision process considered here. To calculate the probabilities for the processes of charge exchange and elastic collision, we need to know the cross sections. Since we are interested in the probabilities for ejection of atmospheric molecules by these processes, we will calculate the cross sections for scattering through all angles greater than a minimum angle, so that the atmospheric particle gains at least enough energy in the process to enable it to escape. We will call this the "escape cross section." A consideration of the dynamics of elastic collisions is necessary now.

Since the de Broglie wavelength for protons of 1300 ev kinetic energy is very much smaller than the dimensions of the smallest atoms, the classical equations of motion provide a very accurate approximation to the correct quantum mechanical equations of motion, and the treatment of collisions will be done here using classical mechanics.

Consider an incident particle of mass m_1 , velocity v and kinetic energy E , and a particle of mass m_2 which will be assumed to be at rest with respect to the moon. We will consider that some interaction takes place between these particles such that the energies and angles of motion of the particles after the collision are as shown in Fig. 5. (Note that we have implicitly used the assumption that the sum of the kinetic energies of the particles before the collision is the same as the sum after the collision, in writing $E - E_f$ for the final velocity of particle 2.) Figure 6 shows the representation of the collision in a coordinate system which always has as its origin the center of

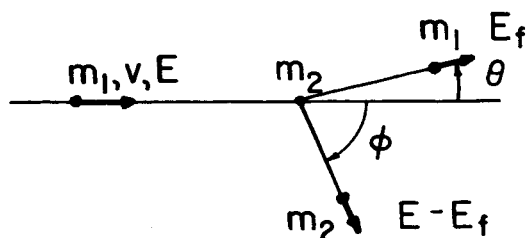


Fig. 5.

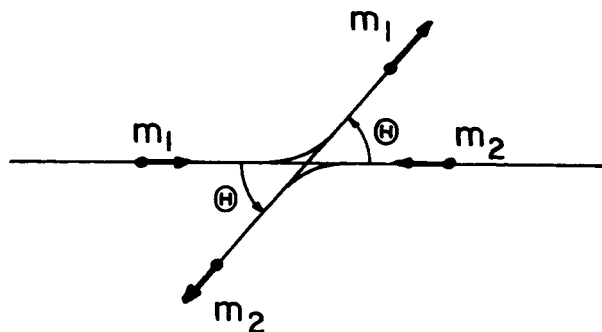


Fig. 6.

Collision coordinate systems.

mass of the two-particle system. It can easily be shown (see Goldstein²⁰) that the following relations are true between the scattering angles:

$$\phi = \frac{\pi}{2} - \frac{\Theta}{2}, \quad \tan \Theta = \frac{\sin \Theta}{\cos \Theta + \frac{m_1}{m_2}}. \quad (3-54)$$

Now let us write the equations expressing conservation of energy and momentum in the following form:

$$\begin{aligned} \sqrt{2m_1 E} &= \sqrt{2m_1 E_f} \cos \Theta + \sqrt{2m_2 (E - E_f)} \cos \phi \\ 0 &= \sqrt{2m_1 E_f} \sin \Theta - \sqrt{2m_2 (E - E_f)} \sin \phi. \end{aligned} \quad (3-55)$$

Squaring and adding to eliminate Θ , we get the energy gained by particle 2 in terms of the angle through which it is scattered:

$$E - E_f = 4E \frac{m_1 m_2}{(m_1 + m_2)^2} \cos^2 \phi. \quad (3-56)$$

It will be convenient to express this in terms of the angle of scattering in the center of mass system:

$$E - E_f = 4E \frac{m_1 m_2}{(m_1 + m_2)^2} \sin^2 \frac{\Theta}{2}. \quad (3-57)$$

Now the energy of escape for particle 2 is $(E - E_f)_{\text{esc}} = \frac{1}{2} m_2 v_{\text{esc}}^2$, where v_{esc} is the escape velocity at the moon's surface. Therefore, the minimum angle Θ through which the scattering can take place and still impart to particle 2 the escape energy is given by

$$\sin \frac{\Theta_{\text{esc}}}{2} = \sqrt{\frac{(m_1 + m_2)^2 v_{\text{esc}}^2}{8m_1 E}}. \quad (3-58)$$

Now the elastic "escape cross section" is given by

$$\sigma_{\text{esc}}^{(\text{el})} = 2\pi \int_{\Theta_{\text{esc}}}^{\pi} \sigma(\Theta) \sin \Theta d\Theta, \quad (3-59)$$

where $\sigma(\Theta)$ is the differential scattering cross section, which depends upon the interaction potential. The above integration does not need to be actually

carried out if we can express the so-called impact parameter b as a function of Θ . The impact parameter is the perpendicular distance between the center of force and the incident particle velocity, which is associated with scattering through a particular angle Θ . If $\Theta = \Theta_{\text{esc}}$, then the elastic "escape cross section" is given by

$$\sigma_{\text{esc}}^{(\text{el})} = \pi b^2(\Theta_{\text{esc}}) . \quad (3-60)$$

Now for the light atoms of H and He, the above cross sections are very nearly equal to the gas kinetic transport cross sections, given by Jeans.²⁶ For heavier atoms, however, $\sigma_{\text{esc}}^{(\text{el})}$ may be considerably smaller than the gas kinetic cross section. This is for two reasons: (1) the energy transferred from incident protons is smaller when the mass of the atmospheric particle is larger; (2) the escape energies for heavy particles are higher than for light particles. We will evaluate $\sigma_{\text{esc}}^{(\text{el})}$ for heavy particles in the following way: The impact parameter $b(\Theta_{\text{esc}})$ corresponding to the minimum scattering angle for escape is found using the result of the Rutherford scattering of protons by charged particles of charge Z_{eff} :

$$b = \frac{Z_{\text{eff}} e^2}{2E_0} \cot \left(\frac{\Theta}{2} \right) , \quad (3-61)$$

where E_0 is the incident kinetic energy minus the kinetic energy of a particle of mass (m_1+m_2) moving with the center of mass (it is the incident energy for the problem in the center of mass coordinates). In terms of the incident energy,

$$E_0 = \frac{m_2}{m_1+m_2} E .$$

The formula which we want is therefore

$$b = \frac{Z_{\text{eff}} e^2 \cot \frac{\Theta}{2}}{\frac{2m_2}{m_1+m_2} E} , \quad (3-62)$$

so that

$$\begin{aligned} b(\Theta_{\text{esc}}) &= \frac{Z_{\text{eff}} e^2 (m_1+m_2)}{2m_2 E} \sqrt{\frac{8m_1 E}{(m_1+m_2)^2 v_{\text{esc}}^2} - 1} \\ &= \frac{Z_{\text{eff}} e^2}{m_2 E} \sqrt{\frac{2m_1 E}{v_{\text{esc}}^2} - \left(\frac{m_1+m_2}{2} \right)^2} . \end{aligned}$$

Since

$$E \gg \frac{1}{2m_1} \left(\frac{m_1 + m_2}{2} \right)^2 v_{esc}^2 ,$$

$$b(\Theta_{esc}) = \frac{Z_{eff} e^2}{m_2 v_{esc}} \sqrt{\frac{2m_1}{E}} . \quad (3-63)$$

In this formula, $Z_{eff} e$ is the effective charge of the atmospheric atom for the scattering, i.e., the nuclear charge Ze minus the charge of the electrons which screen the nucleus. If the deflection angle Θ is small (and so Θ is also small), the force which deflects the incident proton will be approximately given by the force due to a point charge $Z_{eff} e$ at the center of the atom, where Z_{eff} is defined by

$$v(r) \Big|_{r=b} \equiv \frac{Z_{eff} e^2}{b} , \quad (3-64)$$

and where $v(r)$ is the electrostatic potential energy. For heavy atoms, this potential energy can be calculated, using the Thomas-Fermi atomic model. We have

$$Z_{eff}(b) = \frac{bv(b)}{e^2} = ZX[x(b)] , \quad (3-65)$$

where $X(x)$ is the solution of the problem

$$x^{1/2} \frac{d^2 x}{dx^2} = x^{3/2}, \quad x(0) = 1, \quad x(\infty) = 0 , \quad (3-66)$$

and is graphed on page 364 of Ref. 37. Also,

$$x(r) = 2(4/3\pi)^{2/3} Z^{1/3} \frac{r}{a_0} , \quad (3-67)$$

where $a_0 = 5.29 \times 10^{-9}$ cm is the first Bohr radius. Since

$$r = \frac{1}{2} \left(\frac{3\pi}{4} \right)^{2/3} \frac{a_0}{Z^{1/3}} x ,$$

and

$$b(\theta_{esc}) = \frac{ZX\{x[b(\theta_{esc})]\}e^2}{m_2 v_{esc}} \sqrt{2m_1/E} ,$$

we have

$$x(x) = \frac{1}{2} \left(\frac{3\pi}{4} \right)^{2/3} \frac{a_0 m_2 v_{esc}}{Z^{4/3} e^2} \sqrt{\frac{E}{2m_1}} x . \quad (3-68)$$

This equation has been solved graphically for x in the case of Ar, Kr and Xe. The results are given in Table 4 along with the gas kinetic cross sections for the light molecules which are given by Jeans²⁶ and Loeb.³⁸

TABLE 4

CROSS SECTIONS FOR ELASTIC COLLISION AND
CHARGE EXCHANGE WITH 1300 ev PROTONS

Gas	$\sigma^{(el)},$ (cm ²)	$\sigma_{esc}^{(el)},$ (cm ²)	$\frac{\sigma_{esc}^{(el)}}{\sigma^{(el)}}$	$\sigma^{(cx)},$ (cm ²)	$\sigma_{esc}^{(cx)},$ (cm ²)
H	4.5x10 ⁻¹⁶ *	4.5x10 ⁻¹⁶	1	2x10 ⁻¹⁵	2x10 ⁻¹⁵
He	3.7x10 ⁻¹⁶	3.7x10 ⁻¹⁶	1	1.8x10 ⁻¹⁷	1.8x10 ⁻¹⁷
H ₂ O	5.8x10 ⁻¹⁶	5.8x10 ⁻¹⁶	1	2x10 ⁻¹⁵ **	2x10 ⁻¹⁵
Ar	10.5x10 ⁻¹⁶	3.2x10 ⁻¹⁷	0.030	4.4x10 ⁻¹⁶	1.3x10 ⁻¹⁷
Kr	13.5x10 ⁻¹⁶	2.5x10 ⁻¹⁷	0.018	10.6x10 ⁻¹⁶	1.9x10 ⁻¹⁷
Xe	18.7x10 ⁻¹⁶	2.1x10 ⁻¹⁷	0.011	7.9x10 ⁻¹⁶	8.7x10 ⁻¹⁸

*This is an estimate; it is intermediate between the values for He and H₂, given by Jeans.

**This value was adopted as the same as that for H.

3. Charge Exchange

Collisions which result in exchange of charge are not elastic, but the change in internal energy of the colliding particles is only about 2 ev for

protons in Ar and less than 1 ev for protons in Kr or Xe (see Massey and Burhop³⁶). This produces a very small effect dynamically when the incident proton energy is 1300 ev. Therefore, we can treat these collisions as approximately elastic. For the light atoms of hydrogen and helium, the charge exchange "escape cross sections" $\sigma_{\text{esc}}^{(\text{cx})}$, are almost the same as the charge exchange cross sections which have been measured by Fite, et al.,³⁹ and Keene.⁴⁰ We will take the charge exchange cross section for water vapor to be the same as that measured for hydrogen by Fite, et al. Charge exchange cross sections for Ar, Kr, and Xe have been measured by Hasted.⁴¹ Although these would be expected to be larger than the charge exchange "escape cross sections," it is difficult to say what the ratio is exactly. Therefore, we will take the ratio of "escape cross section" to the measured cross section to be the same for both charge exchange and elastic collisions:

$$\sigma_{\text{esc}}^{(\text{cx})} = \sigma^{(\text{cx})} \frac{\sigma_{\text{esc}}^{(\text{el})}}{\sigma^{(\text{el})}}. \quad (3-69)$$

We have listed the measured values of cross sections along with the values for "escape cross sections" used by us in Table 4. Although we have called these cross sections "escape cross sections" the probability of the particles hitting the moon and failing to be ejected from the lunar atmosphere must be taken into consideration; this is done in Section IV.

H. PHOTOIONIZATION AND PHOTODISSOCIATION IN THE LUNAR ATMOSPHERE

Although we have neglected ionization of atmospheric molecules due to solar protons, we cannot neglect ionization due to solar ultraviolet radiation. At times of quiet sun when the solar corpuscular radiation flux is quite small, the effects of the solar ultraviolet radiation with wavelengths longer than 100Å may become more important than the effects of the solar wind, since the intensity of the radiation in this portion of the spectrum would be expected to be approximately the same during active and quiet solar conditions.

Hinterreger³¹ has measured the solar photon fluxes between 1300 and 60Å at an altitude of 200 km and estimated the fluxes in interplanetary space near the earth. On this basis, he gives total ionization rates for several kinds of gases, among them monatomic hydrogen and helium, by summing the products of photoionization cross sections and particle fluxes over the wavelength bands. He obtains the following results:

$$\text{H: } \Sigma \sigma \Phi_0 = 4.5 \times 10^{-7} / \text{sec}$$

$$\text{He: } \Sigma \sigma \Phi_0 = 10^{-7} / \text{sec.}$$

We have estimated the ionization rate for water vapor by using Hinterreger's flux measurements and averages of the photoionization cross sections measured by Wainfan, et al.,⁴² in the range from 1000 to 300Å and those measured by Astoin, et al.,⁴³ in the range from 300 to 250Å. Our result is $\Sigma\sigma_0 = 2.45 \times 10^{-6}/\text{sec}$. We have estimated the photodissociation rate for water vapor in a similar way. Near the threshold for dissociation, 2400Å, Granath⁴⁴ found the absorption cross section to be less than 10^{-22} cm^2 . We have therefore neglected the absorption in the wavelength region between the threshold for dissociation and 1800Å. Using the results of Watanabe, et al.,⁴⁵ for the absorption cross section in the wavelength region between 1800 and 1000Å, and the intensity of the solar spectrum given by Detwiler, et al.,⁴⁶ we find $\Sigma\sigma_0 = 1.62 \times 10^{-5}/\text{sec}$. The total probability for removal of water vapor by either photodissociation or photoionization is therefore $\Sigma\sigma_0 = 1.86 \times 10^{-5}/\text{sec}$.

We have also estimated the ionization rate for argon using Hinterreger's flux measurements and the photoionization cross sections measured by Po Lee and Weissler.⁴⁷ Our result is $\Sigma\sigma_0 = 1.95 \times 10^{-6}/\text{sec}$.

Although the ionization thresholds of krypton and xenon occur at longer wavelengths than does that of argon, the photoionization cross sections, which have never been measured, may be greater than that of argon or they may be less. In view of this uncertainty, we will use the same value of photoionization rate for Kr and Xe as we calculated for Ar.

SECTION IV

A SIMPLE MODEL OF THE LUNAR ATMOSPHERE

We now wish to calculate the expected partial densities of various gases on the moon. Their source and escape rates are based on the different processes discussed in the previous sections.

For our purposes, we will use an even simpler model of the lunar atmosphere than the prementioned isothermal model. The model is constructed as follows: the entire atmosphere is considered to be contained between the surface of the moon and a surface at height $h(T)$, where $h(T)$ is the scale height for the particular gas considered. It was shown in Section III that the total number of particles which are associated with an isothermal atmosphere is given by the density at the surface, n_0 , multiplied by the volume included between the lunar surface and the inner surface of a sphere of radius $r_0 + h$, which we called the "scale volume." We will use an average temperature of 300°K in computing the total number of particles which are included in our model atmosphere. This number will be calculated as follows:

$$N = n_0 V ,$$

where n_0 is the average number of particles/cm³ at the lunar surface, and V is the scale volume for an isothermal atmosphere at 300°K . The change of N with time is due to the following processes.

(a) The Net Influx of Particles at the Inner Surface.—Included in this term are the contributions discussed in Section III-F from the diffuse reflection of hydrogen and helium, the supply of water vapor from trapped ice, the production of Ar from the radioactive decay of K^{40} , and the production of Kr and Xe from primeval gases trapped in rocks and released by meteor bombardment. We neglect recombination of atmospheric ions on the lunar surface.

(b) The Net Outflux of Particles at the Surface of the Outer Sphere.—This term includes only the evaporation escape flux discussed in Section III-E. Although the integration was taken over the surface of the moon, the same total flux escapes through any finite surface surrounding the moon.

(c) The Net Rate at Which Particles are Produced Within the Scale Volume.—In this term we take into consideration the removal processes of photodissociation and photoionization, due to solar ultraviolet radiation, and charge exchange and elastic collision processes due to the solar wind,

all of which result either in the ejection of particles from the atmosphere or in the ionization of atmospheric particles. We neglect recombination in the scale volume.

The three kinds of processes are related to the time rate of change of the total number of particles by the equation

$$\begin{aligned} \frac{dN}{dt} &= \oint_{\text{inner surface}} \vec{J}_S \cdot d\vec{s} - \oint_{\text{outer sphere}} \vec{J}_J \cdot d\vec{s} + \iiint_{\text{scale volume}} S dV \\ &= I_1 - I_2 + I_3, \end{aligned} \quad (4-1)$$

where \vec{J}_S is the source strength, \vec{J}_J is the Jeans escape flux and S is the volume production rate. We have abbreviated the three terms, I_1 , I_2 , and I_3 .

A. HYDROGEN AND HELIUM

The calculation of the atmospheric partial densities is well illustrated by the cases of hydrogen and helium. For these gases, I_1 is equal to the proton flux J_p multiplied by the effective capture cross section of the moon. If we assume that the charged particles in the lunar ionosphere form a space charge region close to any portions of the lunar surface which are charged, so that the charge on the moon is shielded a long distance away, the only forces acting on the particles in the solar wind are due to the gravitational attraction of the moon. Let b be the impact parameter corresponding to particles which just graze the moon's surface. From continuity we have

$$J = J_\infty \frac{b^2}{r_0^2},$$

where J_∞ is the solar wind flux far away from the moon. From conservation of energy and angular momentum we find that

$$r_0^2 + \frac{mMG}{\frac{1}{2} m v_\infty^2} r_0 = b^2,$$

so that

$$\frac{b^2}{r_0^2} = 1 + \frac{2mMG}{m v_\infty^2 r_0} = 1 + \frac{v_{esc}^2}{v_\infty^2},$$

where v_∞ is the velocity of the stream a long distance away from the moon. Since $v_{\text{esc}}/v_\infty \simeq 5 \times 10^{-3}$, $b \simeq r_0$, and the effective capture cross section of the moon is just πr_0^2 . Therefore,

$$I_1 = J_p \pi r_0^2 . \quad (4-2)$$

(We have neglected the contribution to the number of neutral hydrogen atoms in the atmosphere from dissociation of H_2O , which is negligible in comparison to the solar wind contribution.)

I_2 is equal to the evaporation escape flux multiplied by the total surface area of the moon:

$$I_2 = 4\pi r_0^2 \left[\frac{n_0 \bar{c}}{4} e^{-r_0/h} \left(1 + \frac{r_0}{h} \right) \right] . \quad (4-3)$$

The term I_3 is negative for neutral particles. Its magnitude is equal to the "scale volume" V multiplied by β , the fraction of the particles in the atmosphere which are exposed to the sun, multiplied by the average volume rate of all processes which eject hydrogen or helium from the atmosphere:

$$I_3 = -\beta V [n_0 J_{\text{esc}}^{(\text{cx})} \gamma + n_0 J_{\text{esc}}^{(\text{cx})} \gamma + n_0 \sum \sigma \Phi_0] . \quad (4-4)$$

The fraction β will be taken to be equal to 1 for hydrogen and helium. As we stated in Section III-B, we expect the density of hydrogen atoms at the surface to be fairly constant over the surface. As shown in Fig. 4, most of the scale volume, which may be considered to be the volume contained between the surface and the scale height, is exposed to the sun in the case of hydrogen and helium. Therefore, we can take $\beta \simeq 1$ for hydrogen. The value of β for helium is expected to be even more nearly one, since the net rate at which helium atoms can migrate to the dark side after being formed on the sunlit side is considerably smaller than the net rate for hydrogen atoms.

The three contributions to I_3 are due to elastic collision and charge exchange with solar wind protons and to photoionization and photodissociation. The γ -factor is the result of averaging the rate of escape of ejected particles over the entire exposed lunar atmosphere, taking into account the geometric probabilities of ejected particles striking the moon and not escaping. The distribution of scattered intensity with scattering angle for elastic collisions and charge exchange which has been measured (see Massey and Burhop³⁶) shows that the vast majority of incident particles are scattered through very small angles. Since the exchange of energy in charge exchange is small com-

pared to the total kinetic energy of the particles, the dynamics of those processes would be expected to be almost the same as for elastic collisions, as mentioned previously. When the incident particles are deflected at very small angles θ , the struck particles are scattered at angles very close to 90° , since

$$\frac{\sin \theta}{\cos \theta} = \frac{\sin 2\phi}{\frac{m_1}{m_2} - \cos 2\phi}$$

(see Section III-F) and $m_1/m_2 \leq 1$. Most of the particles in the atmosphere are, therefore, scattered nearly at right angles to the incident solar wind direction, so they have a geometric probability of escape close to 1 near the portion of the atmosphere roughly above the subsolar point, and a geometric probability of escape of about $1/2$ near the limbs. The geometric probability averaged over the exposed part of the model atmosphere is therefore between 1 and $1/2$. γ as a function of the molecular weight would be expected to approach 1 as the molecular weight became very small, and to approach $1/2$ as the molecular weight became very large. For hydrogen and helium we will, therefore, use $\gamma = 1$, and we will use $\gamma = 1/2$ for the remaining gases. Using the values of \bar{c} , h , and $\pi r_0^2/V$ taken from Table 1, the values of J and J_p taken from Table 3, the values of σ_{esc} taken from Table 4, and the values of $\sum \sigma \phi_0$ taken from Section III-G, we can proceed to calculate the time rate of change of the atmospheric density of hydrogen and helium as follows:

$$\frac{dN}{dt} = V \frac{dn_0}{dt} = I_1 - I_2 + I_3 ,$$

or

$$\begin{aligned} \frac{dn_0}{dt} = & \frac{\pi r_0^2}{V} J_p - \frac{\pi r_0^2}{V} n_0 \bar{c} e^{-r_0/h} \left(1 + \frac{r_0}{h}\right) \\ & - \beta n_0 (J \sigma_{esc}^{(el)} \gamma + J \sigma_{esc}^{(cx)} \gamma + \sum \sigma \phi_0) . \end{aligned} \quad (4-5)$$

Table 5 gives the numerical values for use in Eq. (4-5). For both hydrogen and helium, the primary escape mechanism is seen to be the evaporation of the tail of the velocity distribution, which does not depend upon assumptions concerning the scale volume, as previously noted. This is expected, since the thermal velocities are comparable to the escape velocity. Given any initial density, the above differential equations govern the time rate of change of the density at the lunar surface according to our model. For $J = 10^9$ part/cm²sec, it is seen that the time constant for approach to an equilibrium value is 7.7×10^3 sec, or about two hours, for hydrogen, and 2.5×10^4 sec, or about seven hours, for helium. The equilibrium values are

TABLE 5

NUMERICAL VALUES FOR EQ. (4-5)

$$\frac{dN}{dt} = v \frac{dn_0}{dt} = I_1 - I_2 + I_3 = \text{particles/sec}$$

$$I_1 = \oint_{\text{inner surface}} \vec{J}_g \cdot d\vec{s} = \text{source strength}$$

$$I_2 = \oint_{\text{outer surface}} \vec{J}_j \cdot d\vec{s} = 4\pi r_0^2 \left[\frac{n_0 \bar{c}}{4} e^{-r_0/h} \left(1 + \frac{r_0}{h} \right) \right] = \text{Jeans escape flux}$$

$$I_3 = \iiint_{\text{scale volume}} S dv = -\beta V n_0 [\gamma J(\sigma_{\text{esc}}^{(el)} + \sigma_{\text{esc}}^{(cx)}) + \Sigma \sigma \Phi_0] = \text{solar wind and solar photon flux}$$

Gas	V	I ₁	I ₂	I ₃
H	1.25x10 ²⁶ cm ³	Jx8.85x10 ¹⁶	n ₀ x1.63x10 ²²	-n ₀ (Jx3.06x10 ¹¹ +5.62x10 ¹⁹)
He	1.79x10 ²⁵ cm ³	Jx6.65x10 ¹⁵	n ₀ x7.19x10 ²⁰	-n ₀ (Jx6.95x10 ⁹ +1.79x10 ¹⁸)
H ₂ O	3.40x10 ²⁴ cm ³	5.71x10 ²²	n ₀ x1.78x10 ¹³	-n ₀ (Jx2.19x10 ⁹ +3.16x10 ¹⁹)
Ar	1.36x10 ²⁴ cm ³	7.23x10 ²²	n ₀ x4.64x10 ³	-n ₀ (Jx1.53x10 ⁷ +1.33x10 ¹⁸)
Kr	6.34x10 ²³ cm ³	1.07x10 ¹⁷	Small compared to other terms	-n ₀ (Jx6.98x10 ⁶ +6.20x10 ¹⁷)
Xe	4.13x10 ²³ cm ³	7.61x10 ¹⁴	Small compared to other terms	-n ₀ (Jx3.10x10 ⁶ +4.03x10 ¹⁷)

thus attained rather quickly; they are given for $J = 10^9$ part/cm² sec as follows:

$$\text{H: } n_0 = 5.30 \times 10^3 \text{ atoms/cm}^3$$

$$\text{He: } n_0 = 9.13 \times 10^3 \text{ atoms/cm}^3.$$

Thus, although the number density of helium in the solar wind is likely to be from 1 to 2 orders of magnitude less than that of hydrogen, the evaporation of the helium provides an escape rate of from 1 to 2 orders of magnitude slower than that of hydrogen, so the equilibrium number densities are about the same. The larger scale volume of hydrogen also adds to this effect.

Since the evaporation flux is of predominant importance for the escape of these light gases, we may also estimate the average number density at the lunar surface by equating the number of particles escaping/cm²/sec from some particular place on the lunar surface due to the evaporation, to the influx $J_p \cos \Theta$, where Θ is the angle which the sun makes with the zenith at that place. Therefore,

$$n(\Theta) = \begin{cases} \frac{J_p \cos \Theta \exp\left(\frac{mgr_0}{kT(\Theta)}\right)}{\sqrt{\frac{kT(\Theta)}{2\pi m}} \left(1 + \frac{mgr_0}{kT(\Theta)}\right)}, & \text{for } 0 < \Theta < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} < \Theta < \pi \end{cases} \quad (4-6)$$

This formula implies a detailed balancing at every point on the surface, which cannot be the case. Because of the long flight paths of particles emitted from the surface on the day side of the moon, there will always be more particles near the terminator, $\Theta = \pi/2$, and on the night side than given by the above formula. The value of n given by the above formula is, therefore, too high in the region near $\Theta = 0$, since the only input is the solar proton flux $J_p \cos \Theta$. The value predicted at $\Theta = 0$ is:

$$n(0^\circ) = 2.17 \times 10^4 \text{ atoms/cm}^3$$

for H. Applying the same reasoning in the case of helium gives

$$n(0^\circ) = 3.67 \times 10^4 \text{ atoms/cm}^3.$$

These values are about four times higher than those gotten with our simple model, where migration of atoms was taken (implicitly) to be efficient

enough that the density was uniform over the entire surface. To see which point of view is more justified, let us assume that an atom of hydrogen and of helium is emitted from the lunar surface at an angle of 45° with the horizontal plane and with the mean speed characteristic of the temperature T . As shown in Fig. 3, the angular ranges of hydrogen for temperatures between 200°K and 400°K , which are typical of the day side, are enough longer than those for temperatures between 120° and 200° to enable migration to take place fairly readily from the day side to the night side. Therefore, the density of hydrogen would be expected to be fairly uniform over the entire surface, as assumed in our simple model. On the other hand, the curve of angular range versus temperature for helium is not nearly as steep, indicating that the net migration of helium atoms from the day side to the night side is not as efficient, resulting, therefore, in a somewhat higher density on the day side.

B. WATER VAPOR

The term I_1 is equal to the equivalent source strength for the entire surface given in Section III-F multiplied by the lunar surface area:

$$I_1 = J_S 4\pi r_0^2 . \quad (4-7)$$

I_2 and I_3 have the same form as for hydrogen and helium. Although the scale height is larger on the sunlit side, so that a slightly larger fraction than $1/2$ of the scale volume is exposed to the sun, we expect somewhat higher densities for water vapor and the heavier gases on the dark side. We will, therefore, use the value $\beta = 1/2$, in the term I_3 and use the value $1/2$ for the γ -factor for water vapor. The largest contribution is seen in Table 5 to be the ionization and dissociation. The contributions due to the solar wind are somewhat smaller. The time constant is 10^5 sec, or about 28 hours. The equilibrium value is $n_0 = 1.7 \times 10^3/\text{cm}^3$ for $J = 10^9$ part/ cm^2 sec. The corresponding partial pressure of water vapor in regions where the temperature is about 120°K is about 2×10^{-14} mm Hg. Thus, the partial pressure is considerably lower than that needed to stop the net evaporation (according to Vestine²⁷ the equilibrium vapor pressure of water at 120°K is about 5×10^{-12} mm Hg), and we are justified in using the evaporation rate which we used.

C. ARGON

The term I_1 has the same form as for water vapor. We neglect I_2 , since it is much smaller than I_3 . The time constant for approach to equilibrium is 10^6 sec, or about 11.6 days, and the equilibrium value is $n_0 = 5.4 \times 10^4/\text{cm}^3$ at $J = 10^9$ part/ cm^2 sec.

D. KRYPTON

The time constant is about the same as for argon, about 11.6 days, and the equilibrium value is $n_0 = 1.7 \times 10^{-2}/\text{cm}^3$.

E. XENON

The time constant is also about 11.6 days, and the equilibrium value is $n_0 = 1.87 \times 10^{-3}/\text{cm}^3$. We see that krypton and xenon comprise a very small portion of the lunar atmosphere.

F. THE VARIATION WITH THE STRENGTH OF THE SOLAR WIND

The composition of the lunar atmosphere will be expected to vary considerably with the strength of the solar wind. To see this, consider the following: in the case of hydrogen and helium, the predominant escape mechanism is the evaporation of the tail of the velocity distribution, while removal by solar corpuscular streams and ultraviolet radiation is negligible in comparison. For water vapor, argon, krypton and xenon, the situation is reversed: the evaporation term is negligible in comparison to the other terms. For hydrogen and helium the source is the solar corpuscular flux itself, while the sources of the other gases lie within the moon or on its surface. Since the time constants for approach to equilibrium of all of these constituents are less than twelve days, the densities, which depend upon the solar wind flux J , would be expected to change in response to a change in J in a correspondingly short time. Figure 7 shows the equilibrium values of density for the constituents whose densities are greatest, Ar, He and H, as a function of J . For values of J much below $10^9/\text{cm}^2 \text{ sec}$, the main constituent is Ar, at a density of about $5.5 \times 10^4/\text{cm}^3$. For values of J above $10^9/\text{cm}^2 \text{ sec}$ the densities of H and He increase with J while the density of Ar decreases, since the effects of the solar wind in sweeping it away begin to predominate over the photoionization. For values of J above about $10^{11}/\text{cm}^2 \text{ sec}$, the density of H increases more slowly with J , and approaches a limit at about $2.9 \times 10^5/\text{cm}^3$, when the increase of hydrogen accretion from the solar wind with increasing J is just balanced by the sweeping away effect of the solar wind. The density of helium continues to increase with increasing J until J reaches about $10^{12}/\text{cm}^2 \text{ sec}$, when it begins to level off and approach the limit $9.6 \times 10^5/\text{cm}^3$.

We have assumed that the flux of ultraviolet radiation is not affected by the level of solar activity, while the corpuscular streams are. Since most of the ionizing and dissociating effects are caused by radiation of wavelengths greater than 100\AA , this is justified if the short wavelength region whose flux is enhanced during periods of solar activity lies below 100\AA , as indicated by Shaw.⁴⁸

We see from Fig. 7 that the minimum density of the lunar atmosphere is about 5.5×10^4 particles/cm³, during conditions of very small solar flux. We must emphasize again, that the representation of the partial densities in the lunar atmosphere in Fig. 7 is incomplete, since we have omitted the gases CO₂ and SO₂. Our simple atmospheric model may be thought of, therefore, as a minimal model of the lunar atmosphere.

G. THE LUNAR SURFACE CHARGE AND CHARGED PARTICLES IN THE LUNAR ATMOSPHERE

Because of the photoionization of atmospheric particles and charge exchange with solar protons, we expect that charged particles will be present in the lunar atmosphere. These charged particles would be able to recombine on the lunar surface if the surface had no repulsive potential. However, there are several ways in which the surface may become charged. On the sunlit side of the moon, the predominant effect is expected to be photoemission of electrons caused by solar ultraviolet radiation. Since this effect tends to charge the surface to a positive potential, we expect the surface to be positively charged over most of the sunlit side of the moon. Therefore, an atmospheric ion cannot strike the surface unless its kinetic energy is greater than the surface potential. Consequently, a negative space charge region will form close to the surface. The potential of the surface is determined by the condition that the number of electrons emitted per unit area and time due to photoemission must equal the difference between the random electron current to a unit area from the space charge region and the solar wind, and the current of ions from the solar wind and those from the atmosphere which have energy greater than the surface potential.

Near the limbs of the moon the photoemission of electrons should be smaller than at the subsolar point, and it is possible for the lunar surface to acquire a negative potential because the random current of electrons from the solar wind and atmosphere is larger than that due to positive ions.

Therefore, if the moon is a poor conductor of electricity, we expect most of the sunlit side to be positively charged and some of the shaded side to be negatively charged. This is justified by the results of Gibson⁴⁹ concerning radio measurements of the properties of the lunar surface. Assuming a dielectric constant $K = 5$, he finds the conductivity of the top few millimeters of the moon's surface corresponding to the measured attenuation to be 1.16×10^{-2} mho/meter. Therefore we expect that the lunar surface will be able to maintain a substantial potential gradient with reasonably small surface currents.

The positive surface charge has another effect on the charged particles in the atmosphere in addition to hindering their recombination. If a neutral particle is ionized while it is within the shielding distance of the surface, it will be ejected from the atmosphere if the potential in volts at the point where it is ionized exceeds the escape energy for the particle in electron

volts. (Escape energies for particles of atomic weight less than 140 are less than 4 ev.) This has been pointed out by Öpik and Singer.⁵⁰ These authors estimate the screening length for an electron density of about 500/cm³ to be about 400 cm. Since this is so small compared to the scale heights for the gases which we consider, this effect will be ignored here.

We will not attempt to compute the magnitude of the surface potential. If we assume that an atmospheric ion has an energy equivalent to temperatures of 1500°K, which is approximately true for the earth's ionosphere, the surface potential need only be about .2 volts to effectively hinder surface recombination on the sunlit side of the moon. If the surface potential is not at least this high, there will be too few charged particles in the lunar atmosphere to worry about.

Therefore, for simplicity, we will assume that no surface recombination or volume recombination will occur on the sunlit side in the moon's atmosphere. Also, in the following computation of ion density, we will assume that the diffusion of ions in the atmosphere to the limbs of the moon, and their eventual recombination there, is negligible compared to the other processes considered which effect ion removal. This, of course, is not strictly true. Our model, therefore, predicts the maximum number of ions that can be present in the moon's atmosphere, assuming that the only atomic species present are those considered here.

The model we wish to present here for the lunar atmospheric ion density is similar to the model for the lunar neutral particle atmosphere. We state that all the ions associated with the atmosphere are contained in the scale volume V_1 , and have a constant number density, n_{0i} , which will be equal to the surface number density in the real case of an exponentially decaying exosphere.

For this model, we assume an ion temperature of 1500°K. Since the plasma must remain neutral, the scale height for each species is determined by assuming that the effective mass of the ion is half the actual mass to account for the electrical forces caused by the much larger scale heights associated with the electrons. For half the mass and five times the temperature the scale heights for the various ions under consideration are 10 times the scale height of the gases given in Table 1 of Section III-C.

The source of the ions is assumed to be that of the number of ions produced by photoionization plus the number of ions experiencing charge exchange with solar protons which do not escape. This term is expressed by:

$$I_1 = \beta n_0 V [J(\sigma^{(cx)} - \sigma_{esc}^{(cx)}) + \sum \sigma \Phi_0] . \quad (4-8)$$

Here, β is the fraction of the neutral particle scale volume which is not shielded from the sun. We will assume here that for H and He, $\beta = 1$; and for H_2O and Ar, $\beta = 1/2$ as before.

The two processes considered here which cause a removal of ions from the atmosphere are the evaporation of the tail of the velocity distribution and the ejection of ions due to collisions with the solar protons.

The Jean's escape flux is expressed by:

$$I_2 = \beta_i \pi r_o^2 n_{oi} \bar{c}_i e^{-r_o/h_i} \left(1 + \frac{r_o}{h_i}\right) \quad (4-9)$$

where β_i is the fraction of the ionized scale volume which is effected by this term and subscript i denotes a quantity associated with the ions.

The removal of ions by collisions with solar protons is given by:

$$I_3 = \beta_i n_{oi} V_i \sigma_{esc}^{(el)} \quad (4-10)$$

where $\sigma_{esc}^{(el)}$ is given from the results of Section III-G to be:

$$\sigma_{esc}^{(el)} = \pi b^2(\Theta_{esc})$$

where

$$b(\Theta_{esc}) = \frac{Z_{eff} e^2}{m_2 v_{esc}} \sqrt{\frac{2m}{E}} \quad (4-11)$$

Here, m_2 is the mass of the positive ion and m is the mass of the particle which strikes it, which we assume to be a proton. For singly ionized ions, we use $Z_{eff} = 1$ so that

$$\sigma_{esc}^{(el)} = \frac{1.7 \times 10^{-15}}{(M.W.)^2} \text{ cm}^2 \quad (4-12)$$

where M.W. is the molecular weight of the atmospheric ion.

Therefore, our equation becomes:

$$\frac{dN_i}{dt} = V_i \frac{dn_{oi}}{dt} = I_1 - I_2 - I_3$$

or

$$\begin{aligned}
 \frac{dn_{oi}}{dt} &= \beta n_o \frac{V}{V_i} [J(\sigma^{(cx)} - \sigma_{esc}^{(cx)}) + \sum \sigma \Phi_o] \\
 &- \beta_i \frac{\pi r_o^2}{V_i} n_{oi} \bar{c}_i e^{-r_o/h_i} (1 + \frac{r_o}{h_i}) \\
 &- \beta_i n_{oi} J_i \sigma_{esc}^{(el)}
 \end{aligned} \tag{4-13}$$

setting $dn_{oi}/dt = 0$ for equilibrium, we get:

$$\frac{n_{oi}}{n_o} = \frac{\beta \frac{V}{V_i} [J(\sigma^{(cx)} - \sigma_{esc}^{(cx)}) + \sum \sigma \Phi_o]}{\beta_i \left[\frac{\pi r_o^2}{V_i} \bar{c}_i e^{-r_o/h_i} (1 + \frac{r_o}{h_i}) + J_i \sigma_{esc}^{(el)} \right]}. \tag{4-14}$$

Obtaining the desired values from Table 4 and Table 6, we get, for $J = 10^9$:

	$\frac{n_{oi}}{n_o}$	$\frac{n_{oi}}{n_o}$
H^+	6.2×10^{-4}	3.3×10^0
He^+	7.2×10^{-6}	5.9×10^{-2}
H_2O^+	1.8×10^{-3}	3.0×10^0
Ar^+	6.31×10^{-3}	3.41×10^2

The total ion density is $347/\text{cm}^3$ for quiet solar conditions. This result agrees well with that of Elsmore,³ which is about $1000/\text{cm}^3$ for somewhat more active solar conditions.

TABLE 6

ION SCALE HEIGHT, SCALE VOLUME AND ESCAPE CROSS SECTIONS

Ion	Mean Thermal Speed, \bar{c} ($T = 1500^\circ\text{K}$)	Scale Height, h_i ($T = 1500^\circ\text{K}$)	Scale Volume, V_i	V/V_i	β_i	$i\sigma_{\text{esc}}^{(\text{el})}$
H^+	5.61×10^5 cm/sec	15,300 km	2.07×10^{28} cm ³	5.82×10^{-3}	1	1.7×10^{-15} cm ²
He^+	2.82×10^5 cm/sec	3,850 km	7.12×10^{26} cm ³	2.52×10^{-3}	1	1.06×10^{-16} cm ²
H_2O^+	1.33×10^5 cm/sec	856 km	5.12×10^{25} cm ³	6.64×10^{-2}	1/2	5.25×10^{-18} cm ²
Ar^+	8.90×10^4 cm/sec	386 km	1.82×10^{25} cm ³	7.47×10^{-2}	1/2	1.06×10^{-18} cm ²

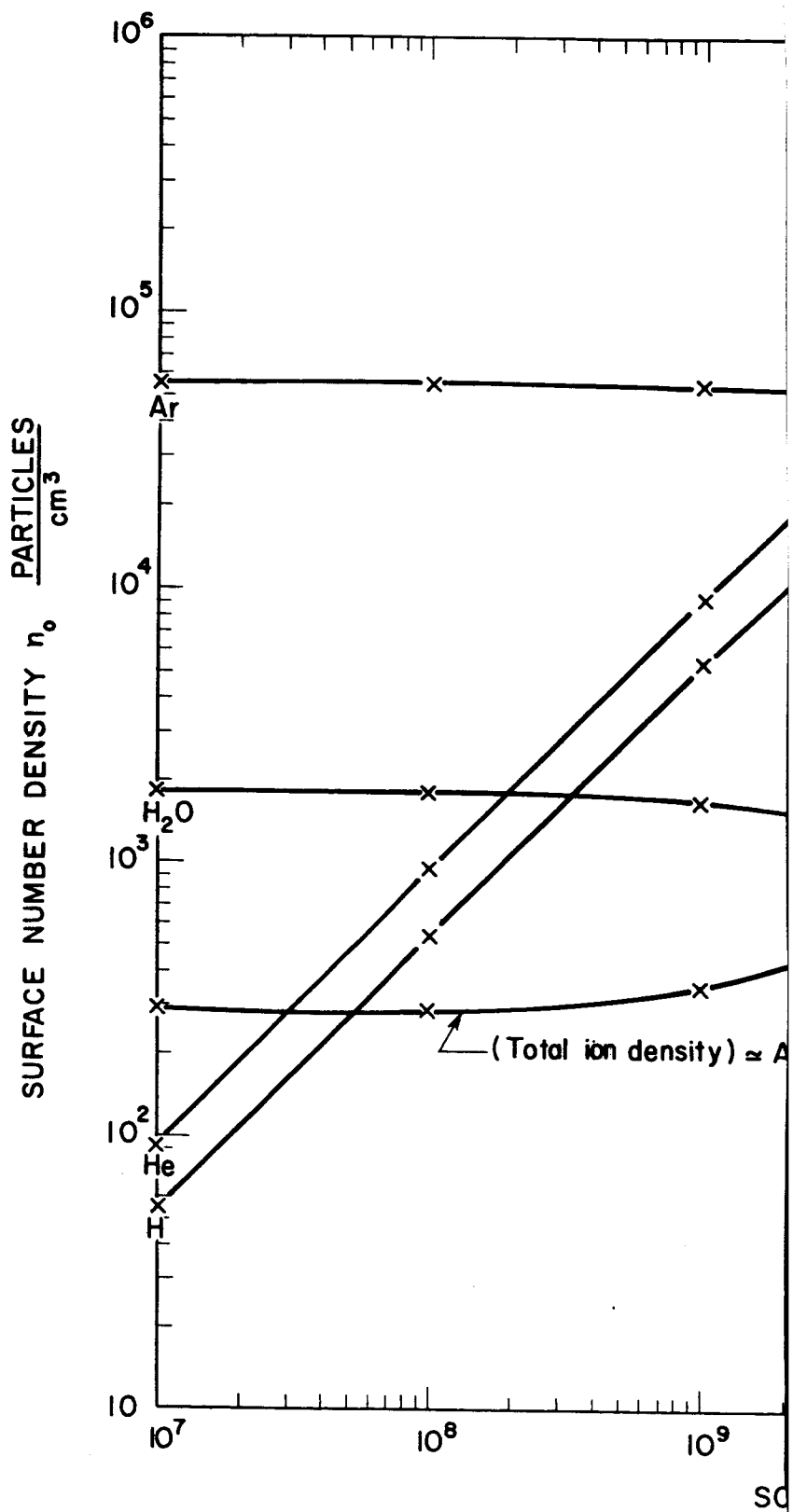
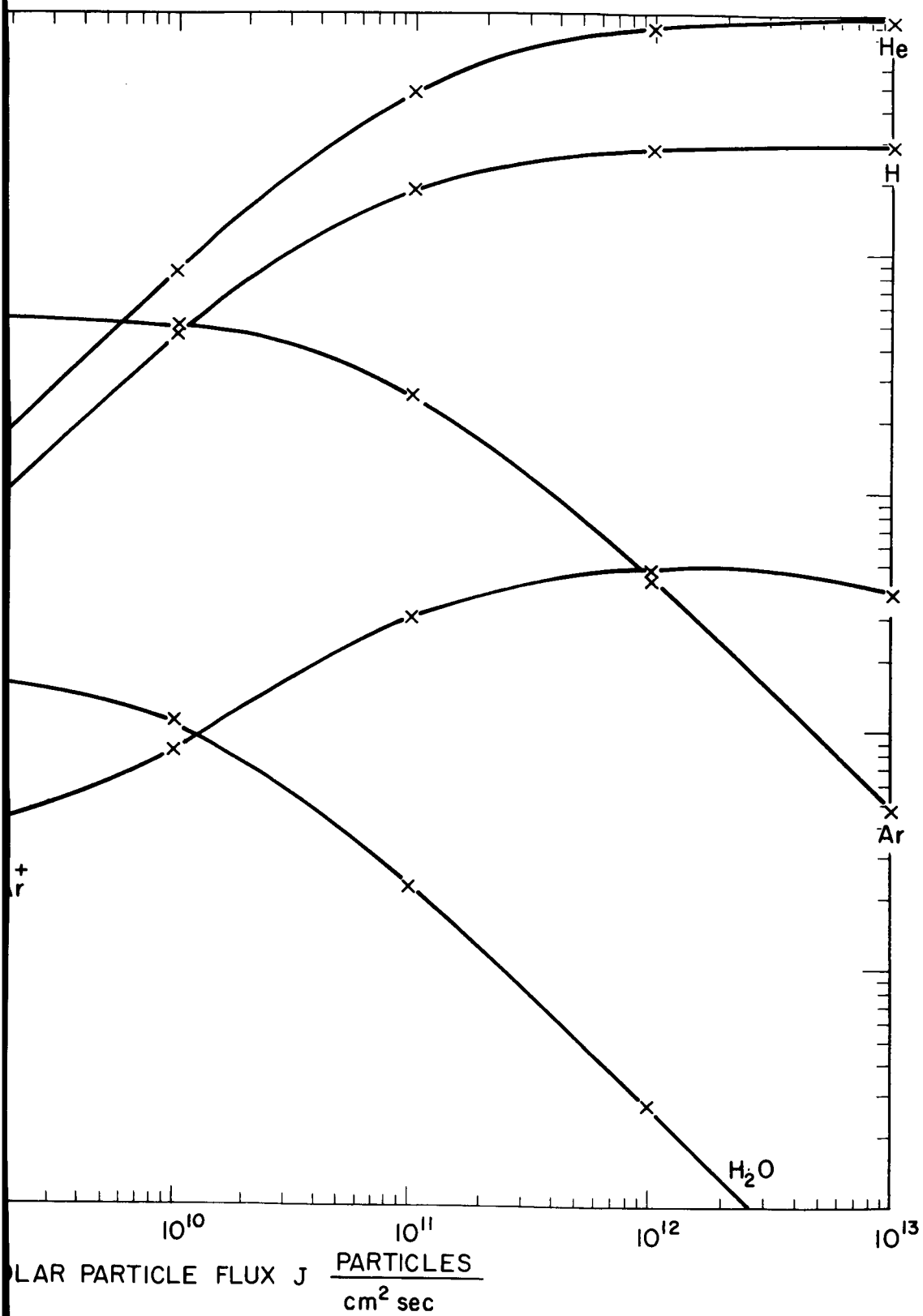


Fig. 7. Lunar model atmosphere.



Surface number density vs. solar particle flux.

SECTION V

DISCUSSION OF THE PROPOSED MEASUREMENT

The experimental determination of the lunar atmospheric structure is a task similar to that required for any planetary atmospheric structure determination. The parameters desired are neutral and charged particle number density, composition, and temperature as a function of altitude, time and latitude. In the case of the terrestrial atmosphere, instruments have been devised to measure all of these quantities to great altitudes. However, the tenuous lunar atmosphere precludes the use of most of the instruments for atmospheric structure measurements on the moon. The most important single deficiency in measurement capability is our inability to measure atmospheric composition on the lunar surface with instruments available today. State-of-the-art flight hardware does very well, with special handling, if it measures partial pressures on the order of the expected maximum total pressure on the moon's surface. Also, this instrumentation is heavy and temperamental and not easily adaptable to the environment expected on the moon.

Therefore, we are limited to the possibilities of neutral and charged particle number density and temperature measurements for initial experiments. Of these, we feel that the neutral particle number density is the most important single measurement. As described in the previous section of this report, the number of charged particles in the lunar atmosphere is probably less than 1% of the number of neutral particles. Therefore it seems important, for initial flights, to measure the neutral density to better understand the orders-of-magnitude with which we are dealing. Once the neutral number density is known, it is possible to view the problem of composition determination more objectively.

A. INSTRUMENTATION

Our initial choice of instrumentation is a cold-cathode discharge gauge (Redhead gauge) with its required electronics. This instrumentation is capable of measuring total neutral number densities from approximately 200 cm^{-3} to $2 \times 10^{10} \text{ cm}^{-3}$. Details of this gauge and the instrument package can be found in Refs. 51 and 52.

Basically, the Redhead gauge is an ionization gauge. It utilizes electrons obtained by a cold-cathode discharge in a crossed electric and magnetic field to ionize the gas. Neutral gas particles enter the gauge through an orifice and tube and are ionized. The resulting ions are collected, producing an ion current which is monitored by an electrometer amplifier. Our instru-

mentation is capable of detecting ion currents from approximately 10^{-6} amps to 10^{-14} amps.

B. CALIBRATION

As mentioned previously, the Redhead gauge to be used for the measurement of the lunar atmospheric neutral number density is an ionization gauge whose ion current is approximately linearly proportional to gas density for any particular gas. The proportionality constant for each particular gas must be experimentally determined, as must the slight variations from linearity for each particular gauge, i.e., the gauge must be calibrated against a known standard. The proportionality constant is called the sensitivity, and is determined primarily by the ionization cross section or ionization probability for a given gas. The performance of this type of gauge is covered in more detail in Ref. 51.

Since we do not know the composition of the moon's atmosphere, it is impossible to calibrate a gauge for the correct gas or mixture of gases. However, we do know that, of the expected gases in the moon's atmosphere, the gauge will have the lowest sensitivity for helium and hydrogen and the highest sensitivity for argon and the residual volcanic gases, with a maximum spread in sensitivity of approximately one order of magnitude. The gauge will be calibrated for as many of these gases and mixtures as possible, so that data received during early flights will be useful when further knowledge of composition is obtained.

It is expected that argon and the volcanic gases will exhibit highest density on the dark side of the moon since they are being evolved from all parts of the moon equally and will be shielded from the solar wind on the dark side. Helium should exhibit highest density on the sunlit part of the moon since it is being produced there. Therefore, the measurement for one lunation should give some indication of the predominant gas, thereby increasing the accuracy of our interpretation of the measurement.

C. MEASUREMENT

The measurement to be made will give us neutral number density versus time at the lunar surface for at least one complete lunation. The expected minimum equatorial number density for our theoretical model is described in Section IV.

The moon's atmospheric neutral number density is interpreted from our measurement of gauge number density by the thermal transpiration equation:

$$n = \sqrt{\frac{T_g}{T_m}} n_g . \quad (5-1)$$

It will be assumed that the gas particles in the gauge will be at the temperature of the gauge (T_g), a measured parameter.

If we assume that the moon's atmosphere is predominantly composed of gas molecules of high molecular weight, such as argon, CO_2 and SO_2 , then the gas temperature outside the gauge (T_m) can be assumed to be at the temperature of the local surface. This can be seen by noting that the range of these molecules, as discussed in Section III, is short enough such that the surface temperature does not vary appreciably over the distance. As an example of this, we see in Section III that argon, at 300°K , has a range of 97.9 km (angular range of 3.20°). Figure 1 gives the variation of surface temperature with angle of rotation with respect to the sun for the lunar equator, and the design package temperature for this condition. As can be seen, an angular range of 3.2° gives a maximum error of approximately 10°K in gas temperature at the point where the surface temperature is 300°K . Of course, near the terminator the temperature varies much more with distance, but the range will also be smaller due to the lower surface temperature.

With the above assumptions, we can plot an expected ratio of gauge number density to atmospheric number density for one lunation (Fig. 8). We divide the measured number density by this ratio for each specific time to obtain the atmospheric number density. All of the above considerations assume that the spacecraft will be on the lunar equator, which will not necessarily be the case. At the time of the experiment, n_g , T_m , and T_g will all be measured, and a true correction factor computed.

D. CONTAMINATION

One problem that confronts us in our measurement is that of the contamination of the moon's atmosphere with gases evolving from the spacecraft. Contamination of the Redhead gauge itself is minimized by evacuating and sealing it on the earth, then breaking the seal after landing on the moon (see Ref. 52). However, the spacecraft itself will be a contributor of gas for some time after being launched from earth, not to mention the retro-rocket contribution of gas upon landing on the moon. The contaminating effect of the outgassing of the spacecraft on our experiment will be minimized by mounting our package on a 15-foot boom, which extends after impact, and orienting the package such that the orifice of our gauge "sees" none of the spacecraft in its hemispherical view. Since the mean-free-path of a gas molecule in the moon's atmosphere is certainly much larger than the dimensions of the spacecraft, we can assume that each molecule will travel in a ballistic trajectory away from the spacecraft. The number of molecules of spacecraft gas having a trajectory which intersects the gauge orifice is assumed negligible, if the gauge is oriented as described above. Also, the total amount of gas which the spacecraft can contribute to the moon's present atmosphere is assumed negligible.

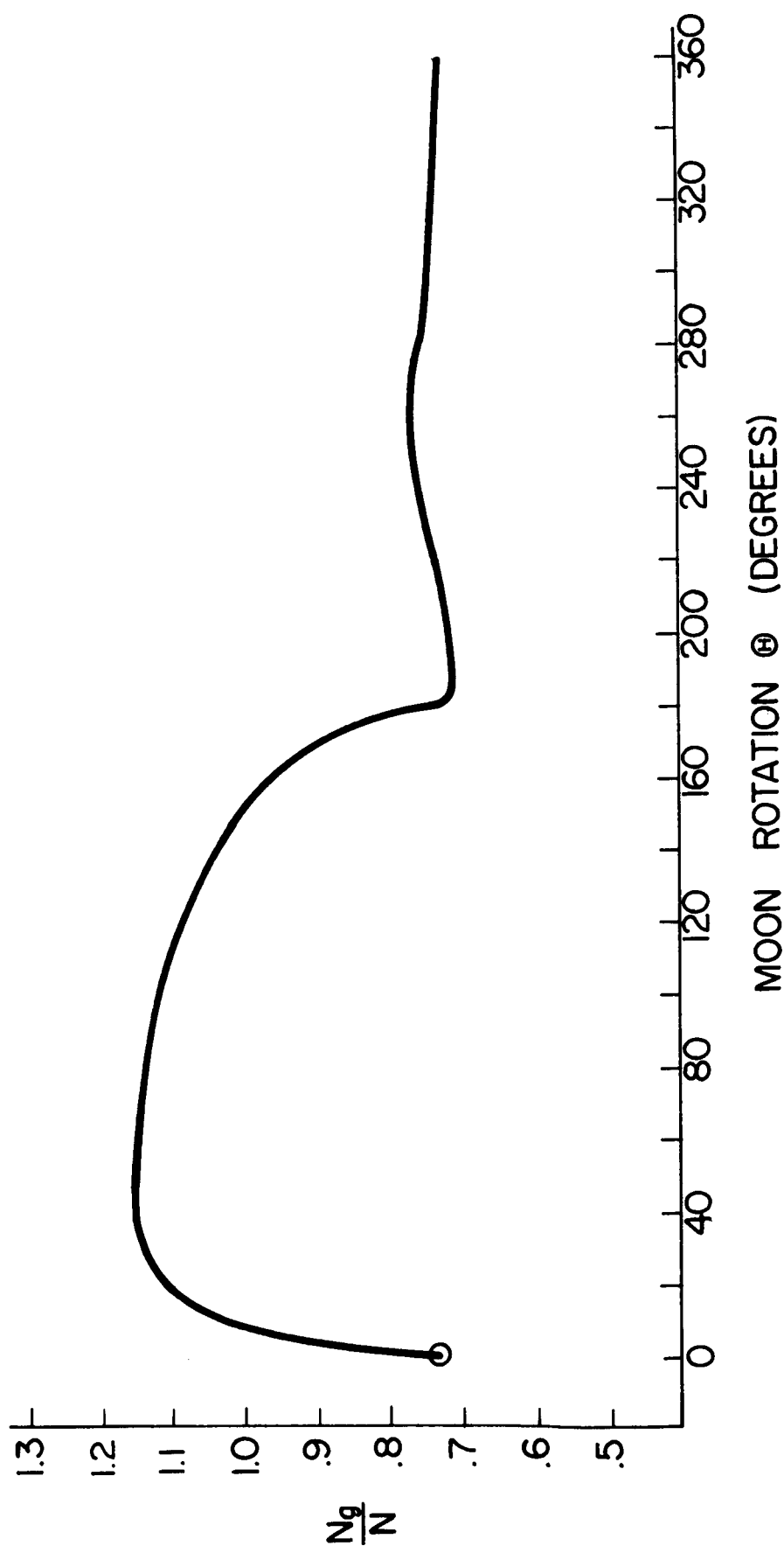


Fig. 8. Ratio of gauge number density to atmosphere number density.

The major problem of contamination is afforded by the retro-rocket motor used during descent to the moon's surface. Table 7 gives the composition and amount of these gases. A quick computation assuming an effective molecular weight of 35 for these gases, a scale height of 50 km, a total weight of gas of 1600 lb yields a surface number density of $6 \times 10^3 \text{ cm}^{-3}$ if the gas is evenly distributed about the moon's surface and escape mechanisms are neglected. Of course, the high initial thermal velocity of these gas molecules will cause a significant percentage of them to escape immediately. The remaining gas will disperse rapidly, becoming a part of the lunar atmosphere. As discussed in Section IV, we feel that the minimum atmospheric density on the moon is about $5.5 \times 10^4 \text{ cm}^{-3}$. Therefore, soon after landing it is expected that the density of the gases from the retro-rocket will be no more than 10% of the atmospheric density.

The prementioned escape mechanisms cause a decay time constant of approximately twelve days or less for gases similar to the retro-rocket gases. Therefore, in the 30-day lifetime of the spacecraft, the amount of retro-rocket gas in the atmosphere will decrease by more than one order-of-magnitude. If surveyors are landed no more often than every 30 days, the effect of the retro-rocket gases on our measurement will be negligible.

E. SOLAR FLUX

Protons from the solar wind will be expected to enter the orifice of the gauge, become neutralized on the inside surfaces, and contribute to the number of neutral particles in the gauge. The contribution to the number density inside the gauge due to solar wind particles will be denoted n_{sg} and will now be calculated, using our previous assumptions concerning the solar wind. If $\langle u \rangle_{avg}$ is the mean velocity of motion of the plasma (500 km/sec), the distribution of the components of velocities normal to the orifice of the gauge, u , in a coordinate system at rest on the moon at some place where η is the angle between the centerline of the orifice and the sun is

$$f(u, \eta) = \left(\frac{m}{2\pi kT} \right)^{1/2} e^{-\frac{m}{2kT} (u - \langle u \rangle_{avg} \cos \eta)^2}, \quad (5-2)$$

where T is the kinetic temperature of the solar wind (10^5°K). If A is the area of the orifice, the number of particles entering the gauge per second is

$$\left[\frac{dn}{dt} \right]_{in} = nA \int_0^\infty u f du = nA \left(\frac{m}{2\pi kT} \right)^{1/2} \int_0^\infty u e^{-\frac{m}{2kT} (u - \langle u \rangle_{avg} \cos \eta)^2} du \quad (5-3)$$

where n is the number density in the solar wind ($20/\text{cm}^3$). Let

$$\beta = \sqrt{\frac{m}{2kT}} (\langle u \rangle_{\text{avg}} \cos \eta) ;$$

the integral can then be evaluated as follows:

$$\begin{aligned} \left[\frac{dn}{dt} \right]_{\text{in}} &= nA \sqrt{\frac{kT}{2\pi m}} \left[e^{-\frac{m}{2kT} \langle u \rangle_{\text{avg}}^2 \cos^2 \eta} \right. \\ &\quad \left. + \sqrt{\frac{\pi m}{2kT}} \langle u \rangle_{\text{avg}} \cos \eta \left(1 + \text{erf} \sqrt{\frac{m}{2kT}} \langle u \rangle_{\text{avg}} \cos \eta \right) \right] \end{aligned} \quad (5-4)$$

Now

$$\sqrt{\frac{m}{2kT}} \langle u \rangle_{\text{avg}} \cos \eta = \frac{3}{2} \frac{\langle u \rangle_{\text{avg}} \cos \eta}{u_{\text{rms}}} = 12.1 \cos \eta ,$$

using values taken from Table 2.

If we take $\eta = 0^\circ$, then we can use the asymptotic form for the error function:

$$1 + \text{erfs} \simeq 2 - \frac{e^{-s^2}}{\sqrt{\pi} s} , \quad \text{for } s \gg 1 . \quad (5-5)$$

Therefore, we have, with

$$\begin{aligned} s &= \sqrt{\frac{m}{2kT}} \langle u \rangle_{\text{avg}} \cos \eta , \\ \left[\frac{dn}{dt} \right]_{\text{in}} &\simeq nA \langle u \rangle_{\text{avg}} = AJ, \end{aligned} \quad (5-6)$$

where J is the solar wind flux.

The number of particles which were contributed by the solar wind which leave the gauge per second is given by the well-known effusion equation:

$$\left[\frac{dn_{\text{sg}}}{dt} \right]_{\text{out}} = n_{\text{sg}} A \sqrt{\frac{kT_{\text{g}}}{2\pi m}} , \quad (5-7)$$

where T_g is the temperature inside the gauge.

When the net number of particles entering the gauge from the solar wind is zero,

$$\left. \frac{dn}{dt} \right|_{in} = \left. \frac{dn_{sg}}{dt} \right|_{out}, \quad \text{or} \quad n_{sg} = \sqrt{\frac{2\pi m}{kT_g}} J. \quad (5-8)$$

If the temperature inside the gauge is 300°K , then $n_{sg} = 1.6 \times 10^4/\text{cm}^3$.

When $\eta = 90^\circ$,

$$\left. \frac{dn}{dt} \right|_{in} = nA \sqrt{\frac{kT}{2\pi m}}. \quad (5-9)$$

so the equilibrium density of particles inside the gauge due to the solar wind is given by

$$n_{sg} = \sqrt{\frac{T}{T_g}} n, \quad (5-10)$$

the ordinary thermal transpiration equation. When $T = 10^5^\circ\text{K}$ and $n = 20/\text{cm}^3$, $n_{sg} = 365/\text{cm}^3$.

In the cases where η is not close to 90° , the contribution of particles inside the gauge due to the solar wind is comparable to that which we expect due to argon, and so the contribution due to the solar wind may be comparable to the total contribution from the neutral particles in the atmosphere. If the flux J is measured independently, we will know n_{sg} during the times when it is large, and corrections to our measurement will be made.

TABLE 7

RETRO ROCKET GASES

<u>Vernier (160 lb)</u>		<u>Main Retro (1450 lb)</u>	
\dot{m}	= 0.1 to 0.33 lb/sec	\dot{m}	= 30 lb/sec
H ₂	32.4%	H ₂ O	12.9%
H ₂ O	19.2	N ₂	9.2
N ₂	31.0	CO	23.7
NH ₃	.001	HCl	17.2
CH ₄	.2	CO ₂	3.6
CO	2.7	H ₂ S	.1
CO ₂	14.5	H ₂	33.3
Burns from 280,000 ft to 13 ft		Burns out between 9000—55,000 ft Ignition ~ 280,000 ft	

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Variation of the Lunar Atmosphere with the Strength of the Solar Wind

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Abstract. A simplified model of the lunar atmosphere is used to calculate average densities of seven different neutral gases and their ionized components, near the moon's surface, as a function of the solar corpuscular flux J . The method of calculation takes into account, in a unified way, the effect of sources of gas in the lunar crust as well as accretion by the solar wind, and the escape of gases by thermal evaporation and photoionization as well as interaction with the solar wind. For $10^7/\text{cm}^2 \text{ sec} < J < 10^9/\text{cm}^2 \text{ sec}$, the main constituents of the lunar atmosphere are argon, constant at about $5 \times 10^6/\text{cm}^3$, and neon, which increases linearly with J . For $10^9/\text{cm}^2 \text{ sec} < J < 10^{11}/\text{cm}^2 \text{ sec}$, argon and water vapor decrease with increasing J , whereas hydrogen, helium, and neon increase. For $10^{11}/\text{cm}^2 \text{ sec} < J < 10^{13}/\text{cm}^2 \text{ sec}$, the densities of argon, neon, hydrogen, and helium take on the approximately constant values (independent of J) of $2 \times 10^6/\text{cm}^3$, $2.8 \times 10^6/\text{cm}^3$, $3.1 \times 10^6/\text{cm}^3$, and $1.5 \times 10^6/\text{cm}^3$, respectively. Hence, the total number density of gas from the sources considered will not exceed $2.1 \times 10^6/\text{cm}^3$, regardless of how strong the solar flux may become. The densities of krypton and xenon will never be larger than $0.38/\text{cm}^3$ and $0.057/\text{cm}^3$, respectively. The only gases that contribute appreciably to the ionosphere are neon and argon.

INTRODUCTION

The purpose of this paper is to present a theoretical calculation of the average number densities of seven neutral gases and their ionized components, near the moon's surface, as a function of the solar corpuscular flux (solar wind). As has been pointed out by Gold [1959], Herring and Licht [1959], Singer [1961], Nakada and Mihalov [1962], and Weil and Barasch [1963], the solar wind has a considerable effect on, and, in fact is, a major source of the lunar atmosphere and ionosphere.

We attempt here to consider, in a unified way, all the major mechanisms that affect the equilibrium number density of the atmosphere near the moon's surface, i.e., the sources of gas from the lunar crust and accretion from the solar wind, and the escape of gases from the moon by thermal evaporation (Jeans' escape mechanism), photoionization, and interaction with the solar wind.

In the theoretical treatment presented, a simplified model of the atmosphere is adopted which assumes that the total number of particles of a given species associated with the atmosphere is contained in the volume between the surface of the moon and a surface at height $h = kT/mg$, the scale height for a gas of par-

ticles of mass m . Therefore, the total number of particles of a given species in the model atmosphere is $N = n_0 V$, where $V = \frac{4}{3}\pi[(r_0 + h)^3 - r_0^3]$ is the 'scale volume.' n_0 is the number density of the gas in question (constant throughout the volume V), which represents the average surface number density for our calculations.

The time rate of change of the number of particles is then given by

$$\frac{dN}{dt} = \iiint s \, dV + \iint_{s_1} J_s \, dS_1 - \iint_{s_2} J_f \, dS_2 \quad (1)$$

The volume integral represents the effects of all the sources and sinks within the scale volume, which includes photoionization and elastic collision and charge exchange with the solar wind ions. The surface integral over S_1 represents the source of gas from the lunar surface into the scale volume. The surface integral over S_2 represents the loss of gas from the surface at $(r_0 + h)$ due to thermal evaporation.

NEUTRAL PARTICLES

This section is devoted to the explanation of the calculation of the integrals of equation 1

for neutral particles in the lunar atmosphere. For the calculations we assume a gas temperature of 300°K, which is approximately the average lunar surface temperature. It is implicitly assumed that the surface number density is low enough so that the mean free path of a gas molecule is larger than the distance it would travel on a trajectory from one position on the moon to another; therefore, the gas will have the temperature of the lunar surface.

Solar wind effect. For the volume integral of equation 1, we assume that the solar wind consists of H^+ , He^+ , Ne^+ , Ar^+ , Kr^+ , and Xe^+ in the relative amounts, X_i , as given in Table 1, adapted from an abundance compilation for the primordial solar system given by *Aller* [1961]. The flux of the i th element in the solar wind is then given by $X_i J$, where J is the total positive ion flux in particles/cm² sec.

Although the solar flux, J , varies with time, a constant value of 500 km/sec for the stream velocity is assumed, so that cross sections for elastic collision and charge exchange may be taken as constants, computed for incident ions moving at 500 km/sec (1250 volts/unit mass).

Elastic scattering. Cross sections for elastic scattering through any angle greater than a minimum angle, such that an atmospheric particle gains at least enough energy in the process to escape ('escape cross sections'), have been calculated by *Hinton and Tausch* [1962] for the heavy atoms of Ar, Kr, and Xe. For scattering of incident particles with mass m_1 and kinetic energy E by atmospheric particles with escape energy, $1/2 m_2 V_{esc}^2$, the result is $\sigma_{e1}' = \pi b_{esc}^2$, where b_{esc} is the impact parameter for Rutherford scattering and is given by

$$b_{esc} \simeq \frac{Z_{eff} e^2}{m_2 V_{esc}} \sqrt{\frac{2m_1}{E}}$$

Z_{eff} is the effective charge used in the Rutherford scattering formula, i.e., the nuclear charge Ze minus the charge of the electrons screening the nucleus. If the deflection angle is small, the force that deflects the incident proton will be approximately given by the force due to a point charge $Z_{eff}e$ at the center of the atom, where $Z_{eff}e$ is defined by

$$[V(r)]_{r=b} \equiv Z_{eff} e^2 / b$$

where $V(r)$ is the electrostatic potential energy.

TABLE 1. Assumed Fractions of Elements in the Solar Wind
Adapted from *Aller* [1961]

Element	X
H	0.86
He	0.14
Ne	4.3×10^{-4}
Ar	6.5×10^{-6}
Kr	1.4×10^{-9}
Xe	10^{-10}

For heavy atoms, this potential energy can be calculated using the Thomas-Fermi atomic model. Therefore, we have

$$Z_{eff}(b) = \frac{b V(b)}{e^2} = Z \chi(x(b))$$

where $\chi(x)$ is the solution of the equation $x^{1/3} d^2 \chi / dx^2 = \chi^{3/2}$, with the condition that $\chi(0) = 1$, $\chi(\infty) = 0$. The solution is graphed by *Leighton* [1959]. Also, $x(r) = 2(4/3\pi) Z^{1/3} r/a_0$, where $a_0 = 5.29 \times 10^{-9}$ cm is the first Bohr radius. Since $r = \frac{1}{2} (3\pi/4)^{2/3} a_0 / Z^{1/3} x$, we now have

$$b_{esc} = \frac{Z \chi(x(b_{esc})) e^2}{m_2 V_{esc}} \sqrt{\frac{2m_1}{E}}$$

where

$$\chi(x) = \frac{1}{2} \left(\frac{3\pi}{4} \right)^{2/3} \frac{a_0 m_2 V_{esc}}{Z^{1/3} e^2} \sqrt{\frac{E}{2m_1}} x$$

This last equation was solved graphically for x for Ar, Kr, and Xe, and the results for $\sigma_{e1}' = \pi b_{esc}^2$ are given in Table 2.

For the atoms of H, He, H₂O, and Ne, for which the Thomas-Fermi model is not applicable, it is assumed that $\sigma_{e1}' \simeq \sigma_{e1}$, where σ_{e1} is the gas kinetic cross section [*Jeans*, 1954], because (1) the average energy transferred in a collision is greater the more nearly equal are the masses of the colliding particles, and (2) the escape energy is lower for lighter particles. The computed and assumed values are given in Table 2.

The rate of escape of particles per second per cubic centimeter of lunar atmosphere is $J \gamma \sigma_{e1}' n_0$, where γ is the geometric probability that a particle receiving enough energy in an elastic collision to enable it to escape will actually escape and not strike the moon and be re-

TABLE 2. Assumed Cross Sections

M	$\sigma_{e1}', \text{ cm}^2$	$\sigma_{ex}(\text{H}^+, \text{M}), \text{ cm}^2$	$\sigma_{ex}(\text{He}^+, \text{M}), \text{ cm}^2$	γ	$\Sigma X\sigma_1, \text{ cm}^2$
H	4.5×10^{-16}	$2 \times 10^{-16*}$	$3 \times 10^{-17} $	1	1.7×10^{-16}
He	3.7×10^{-16}	$2 \times 10^{-17}\dagger$	$7 \times 10^{-16} $	1	1.1×10^{-16}
H ₂ O	5.8×10^{-16}	$2 \times 10^{-16}\dagger$...	0.5	10^{-15}
Ne	10^{-16}	$10^{-16}\dagger$...	0.5	5.0×10^{-17}
Ar	3.2×10^{-17}	$4.4 \times 10^{-16}\S$	$4 \times 10^{-16} $	0.5	4.4×10^{-16}
Kr	2.5×10^{-17}	$1.1 \times 10^{-16}\S$...	0.5	1.1×10^{-16}
Xe	2.1×10^{-17}	$7.9 \times 10^{-16}\S$...	0.5	8.0×10^{-16}

* *File et al.* [1958].

† Extrapolated, from *Stier and Barnett* [1956].

‡ Assumed.

§ Extrapolated, from *Hasted* [1951].

|| *Barnett and Stier* [1958].

emitted into the atmosphere. Since most of the atmospheric particles are scattered nearly at right angles to the incident solar wind direction, $\gamma = 1$ above the subsolar point and $\gamma \simeq 1/2$ above the limbs. The average over the sunlit side of the moon's surface, $\bar{\gamma}$, will be between $1/2$ and 1 for all gases, being about 1 for light gases and about $1/2$ for heavy gases.

Charge exchange. We assume that the sunlit side is positively charged to a few volts, since solar ultraviolet radiation will cause photoemission of electrons from the surface. Because of the repulsive surface potential, ions will not be able to recombine on the surface. Since volume recombination may be assumed negligible [Weil and Barasch, 1963], the ions formed in the atmosphere will be swept away by the solar wind (and the magnetic fields that it carries) before they are able to recombine. An exception is the group of particles that undergo charge exchange and receive enough energy to reach the positively charged surface and recombine there. Assuming arbitrarily that the necessary energy is equal to the escape energy, the cross section for this event is $(1 - \gamma) \sigma_{ex}'$, where σ_{ex}' is the charge exchange 'escape cross section.' Hence, the average cross section for actual removal from the atmosphere by charge exchange is $\sigma_{ex} - (1 - \bar{\gamma}) \sigma_{ex}' \equiv \sigma_1$.

To calculate σ_{ex}' , the authors assumed the relation $\sigma_{ex}'/\sigma_{ex} = \sigma_{e1}/\sigma_{e1}'$, where the σ_{ex} is a measured charge exchange cross section. Computed and assumed values for cross sections and the assumed values for the geometric probability factor are given in Table 2.

The rate of removal of particles per second per cubic centimeter of lunar atmosphere is given by $Jn_0 \Sigma X\sigma_1$, where the sum is over all

elements contained in the solar wind. For practical purposes the only important terms in this summation are those for charge exchange by the ions H^+ and He^+ . Although the reaction $\text{He}^+ + \text{H} \rightarrow \text{He} + \text{H}^+$ is not nearly as important as the reaction $\text{H}^+ + \text{H} \rightarrow \text{H} + \text{H}^+$, the term $Jn_0 X_{\text{H}} \sigma_1$ (He^+, He), for the reaction $\text{He}^+ + \text{He} \rightarrow \text{He} + \text{He}^+$, is about six times larger than the term $Jn_0 X_{\text{H}} \sigma_1$ (H^+, He), for the reaction $\text{H}^+ + \text{He} \rightarrow \text{H} + \text{He}^+$. Therefore, removal of helium from the lunar atmosphere by charge exchange with helium is considerably more effective than by charge exchange with hydrogen, in spite of the abundance difference. For the gases of H_2O , Ne, Ar, Kr, and Xe, we ignore the difference between H^+ and He^+ in charge exchange and replace $\Sigma X\sigma_1$ by σ_1 for protons.

Since charge exchange is generally about an order of magnitude more probable than ionization by impact, the latter process has been ignored.

Photoionization. The photoionization rates for H and He have been estimated by *Hinterreger* [1960]. We have estimated in a similar way the rates of photoionization of argon and neon, and the rates of photoionization and photodissociation of water vapor. The total rates are given in Table 3. The rate of removal of particles per second per cubic centimeter of the lunar atmosphere is then $n_0 \Sigma \sigma \Phi_0$.

The volume integral in equation 1 may, therefore, be calculated as follows:

$$\begin{aligned}
 \iiint s \, dV &= \iiint -\{J(\gamma\sigma_{e1}' \\
 &+ \Sigma X\sigma_1) + \Sigma \sigma\Phi_0\}n_0 \, dV \\
 &= -\beta V\{J(\bar{\gamma}\sigma_{e1}' + \Sigma X\sigma_1) + \Sigma \sigma\Phi_0\}n_0
 \end{aligned}$$

TABLE 3. Rates of Removal by Photoionization and Photodissociation

Gas	$\Sigma\sigma\Phi_0$
H	$4.5 \times 10^{-7}/\text{sec}$
He	$10^{-7}/\text{sec}$
H ₂ O	$1.9 \times 10^{-5}/\text{sec}$
Ne	$3 \times 10^{-7}/\text{sec}$
Ar	$2 \times 10^{-6}/\text{sec}$

where β is the fraction of the particles in the atmosphere that are exposed to the sun assumed to be approximately 1 for H and He and $\frac{1}{2}$ for heavier gases.

Sources of gas from the lunar surface. The integral over the lunar surface S_1 of the source strength J , is the flux of particles of the given species into the lunar atmosphere due to neutralization and diffuse reflection of solar wind ions and to actual sources in the lunar surface crust. We assume that solar wind ions are neutralized when they strike the moon's surface, accommodated to the lunar surface temperature, and re-emitted into the atmosphere, a process first suggested by Gold [1959] for hydrogen. We assume that the gases H, He, Ne, Ar, Kr, and Xe, which are assumed to be present in the solar wind, are supplied in this way. [Other gases, for which the Gold mechanism is uncertain because of chemical combination on the surface (oxygen and nitrogen), were not considered.]

We assume that water vapor is supplied to the lunar atmosphere by evaporation from ice retained on portions of the lunar surface that are permanently shaded from the sun. This possibility has been suggested by Watson *et al.* [1961]. We have estimated the equivalent source strength, based upon the entire lunar surface, to be $J_s = 1.5 \times 10^5$ molecules/cm² sec.

A source for argon in addition to solar wind accretion, namely, radioactive decay of K⁴⁰, has been estimated by Vestine [1958]. After correcting for the decrease in activity over the estimated lifetime of the moon (about twice the half-life of K⁴⁰), we have assumed a source

strength of $J_s = 1.9 \times 10^5$ atoms/cm² sec.

We have also assumed further sources of krypton and xenon from primeval gases trapped in rocks and released by meteor bombardment, on the basis of work of Edwards and Borst [1958]. The equivalent source strengths we have used are $J_s = 2.8 \times 10^{-3}$ atom/cm² sec for krypton and $J_s = 2 \times 10^{-3}$ atom/cm² sec for xenon.

To get the expression for the value of the surface integral, we multiply the solar wind flux XJ by the cross section of the moon, and multiply the internal source strengths by the area of the moon:

$$\iint_{S_1} J_s dS_1 = XJ\pi r_0^2 + J_s 4\pi r_0^2$$

The third integral in equation 1 is the flux of thermally escaping particles over the sphere S_2 at height h . In the first approximation we may calculate this flux at the lunar surface, thus counting in the escape flux some particles that are also counted in the volume integral, i.e., particles that will also interact with the solar wind or undergo photoionization. Assuming the lunar atmosphere to be a classical exosphere, with the lunar surface as its base, we use the formula derived by Jeans [1916] as follows:

$$J_J = \frac{n_0 \bar{c}}{4} e^{-r_0/h} \left(1 + \frac{r_0}{h} \right)$$

where $\bar{c} = \sqrt{8kT/\pi m}$, the mean thermal speed.

The third integral, therefore, becomes

$$\iint_{S_2} J_J dS_2 = \pi r_0^2 n_0 \bar{c} e^{-r_0/h} \left(1 + \frac{r_0}{h} \right)$$

Equation 1 can now be written as follows:

$$\begin{aligned} \frac{dN}{dt} = & V \frac{dn_0}{dt} \\ = & -\beta V \{ J(\bar{\gamma}\sigma_{s1}' + \sum X\sigma_{s1}) + \sum \sigma\Phi_0 \} n_0 \\ & + XJ\pi r_0^2 + J_s 4\pi r_0^2 - \pi r_0^2 n_0 \bar{c} e^{-r_0/h} \left(1 + \frac{r_0}{h} \right) \end{aligned}$$

When J is constant in time, the solution $n_0(t)$ is an exponential which decays or rises from any initial value to the equilibrium value

$$n_0 = \frac{XJ\pi r_0^2 + J_s 4\pi r_0^2}{\beta V \{ J(\bar{\gamma}\sigma_{s1}' + \sum X\sigma_{s1}) + \sum \sigma\Phi_0 \} + \pi r_0^2 \bar{c} e^{-r_0/h} \left(1 + \frac{r_0}{h} \right)}$$

with a relaxation time of

$$\tau = \frac{V}{\beta V \{ J(\bar{\gamma}\sigma_{e1}' + \sum X\sigma_i) + \sum \sigma\Phi_0 \} + \pi r_0^2 \bar{c} e^{-r_0/h} \left(1 + \frac{r_0}{h} \right)}$$

We have used the above method to calculate n_0 as a function of solar flux J for seven different gases. The results are shown in Figure 1 for all except krypton and xenon, for which the equilibrium densities are only $0.38/\text{cm}^3$ and $0.057/\text{cm}^3$, respectively, when the total solar wind flux is as high as $10^{12}/\text{cm}^2 \text{ sec}$.

In the *weak solar wind regime*, from $J = 10^7/\text{cm}^2 \text{ sec}$ to about $J = 10^9/\text{cm}^2 \text{ sec}$, the main constituents of the lunar atmosphere are argon and neon, with argon approximately constant at about $5 \times 10^4/\text{cm}^3$ and neon increasing linearly with J . Hydrogen and helium are also present in appreciable amounts and increase linearly with J , the important removal mechanism being the Jeans escape flux. The amount of water vapor

is approximately constant at about $2 \times 10^3/\text{cm}^3$. The densities of water vapor and argon do not vary appreciably in this regime because the primary sources are contained in the moon and the most important removal mechanism is the solar ultraviolet radiation, assumed to be constant.

In the *intermediate solar wind regime* from $J = 10^9/\text{cm}^2 \text{ sec}$ to $J = 10^{11}/\text{cm}^2 \text{ sec}$, the densities of argon and water vapor begin to decrease, owing to the increasing effectiveness of solar wind removal. The accretion from the solar wind continues to increase the densities of neon, helium, and hydrogen.

In the *strong solar wind regime* from $J = 10^{11}/\text{cm}^2 \text{ sec}$ to $J = 10^{13}/\text{cm}^2 \text{ sec}$, the density of water vapor approaches a linear decrease with

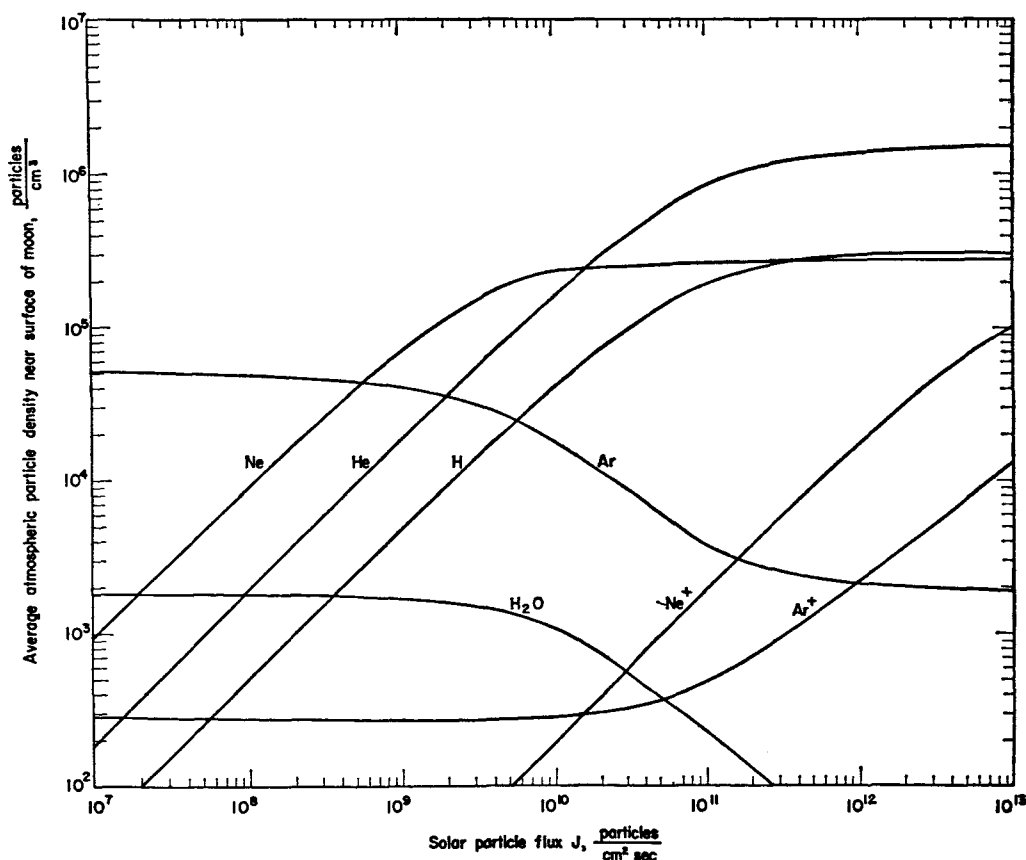


Fig. 1. The average atmospheric particle density near the lunar surface as a function of the solar wind flux.

J , since the solar wind is now the most important removal mechanism. The accretion of argon from the solar wind begins to be important in this regime, and a saturation density of about $2 \times 10^6/\text{cm}^3$ is reached when rates of accretion and removal by solar wind become equal. The densities of neon, hydrogen, and helium saturate similarly at $2.8 \times 10^6/\text{cm}^3$, $3.1 \times 10^6/\text{cm}^3$, and $1.5 \times 10^6/\text{cm}^3$, respectively, for large values of J , independent of how strong the solar flux may become. The total number density of gas from the sources considered here will, therefore, not exceed $2.1 \times 10^6/\text{cm}^3$.

The relaxation times are also functions of J , being relatively long for small values of J and short for large values of J . Hence, the density of lunar atmosphere will be unaffected by fluctuations in J lasting less than a few hours when the solar wind is weak, will be sensitive to almost any variation in J for larger values of J , and will again be insensitive to variations in J when the saturation region is reached.

CHARGED PARTICLES

For ionized particles, the integrals in equation 1 are calculated in a way similar to that for neutral particles. The volume integral consists of two parts: the source of ions from photoionization and charge exchange of neutrals, and the loss of ions swept away by the solar wind:

$$\iiint s dV = \beta V n_0 \left[J \sum X \sigma_2 + \sum \sigma \Phi_0 \right] - V_i n_{0i} J \sigma_3$$

where $\sigma_2 \equiv \sigma_{ex} - \sigma_{ex}'$. The scale volume V_i for ions is calculated by assuming a scale height $h_i = kT_i/m_i g$, where we have chosen the ion temperature to be 1500°K . The effective ion mass m_i to be used here is half the neutral particle mass, because of the very large scale height of electrons and the electric field which tends to establish electrical neutrality. The charged particle scale height, therefore, becomes 10 times the neutral particle scale height, and the scale volume is also considerably larger. Hinton and Tausch [1962] have obtained the formula

$$\sigma_3 = \frac{1.73 \times 10^{-15}}{(\text{M.W.})^2} \text{ cm}^2$$

for the escape cross section for ions by elastic collision with solar wind particles.

The integral over the surface S_1 was assumed to be zero, thus neglecting recombination on the lunar surface. The integral over the surface S_2 is calculated using the Jeans formula and multiplying by half the surface area of the moon, since few ions will exist on the dark side of the moon:

$$\iint_{S_2} J_J dS_2 = 2\pi r_0^2 \left[\frac{n_{0i} \bar{c}_i}{4} e^{-r_0/h_i} \left(1 + \frac{r_0}{h_i} \right) \right]$$

Using equation 1, the requirement that the ion density be constant in time becomes

$$V_i \frac{dn_{0i}}{dt} = \beta V n_0 [J \sum X \sigma_2 + \sum \sigma \Phi_0] - V_i n_{0i} J \sigma_3 - \frac{\pi r_0^2}{2} n_{0i} \bar{c}_i e^{-r_0/h_i} \left(1 + \frac{r_0}{h_i} \right) = 0$$

so that the equilibrium value is

$$n_{0i} = \frac{\beta V n_0 [J \sum X \sigma_2 + \sum \sigma \Phi_0]}{V_i J \sigma_3 + \frac{\pi r_0^2}{2} \bar{c}_i e^{-r_0/h_i} \left(1 + \frac{r_0}{h_i} \right)}$$

The calculations show that the only gases that contribute appreciably to the ionosphere are neon and argon. The results are shown in Figure 1, with the neutral gas results. For values of $J < 10^{10}/\text{cm}^2 \text{ sec}$, the ionosphere is primarily Ar^+ , at a density of about $300/\text{cm}^3$. For $10^{10}/\text{cm}^2 \text{ sec} < J < 10^{11}/\text{cm}^2 \text{ sec}$, Ne^+ becomes the predominant constituent of the ionosphere, its density increasing linearly with J . In deriving these results, we have neglected the effect of the magnetic field carried by the solar wind, which will be very effective in sweeping away the ions in the lunar ionosphere. For larger values of J than about $10^{10}/\text{cm}^2 \text{ sec}$, the ion density may well decrease with increasing J . We have also neglected the removal, by electrostatic repulsion, of ions formed within the screening length near the moon's positively charged surface, since the number of ions formed within the screening length is small compared with the number formed within the scale volume. As given by Öpik and Singer [1960], the ratio of scale height to screening length for Xe, the heaviest molecule considered, is 10^3 . The time constant for removal of the entire exosphere for this mechanism is approximately 10^8 years, assuming no sources of

gas, whereas the time constant, or relaxation time, for the mechanisms considered in this paper is of the order of 12 days for the heaviest molecules.

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